

Name \_\_\_\_\_

Score \_\_\_\_\_

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

Part I. (18 pts) Do three (3) of the following four problems:

1. Find the Laplace transform of  $f(t) = \begin{cases} t+1 & 0 \leq t < 3 \\ -2 & 3 \leq t < \infty \end{cases}$
2. Find the Laplace transform of  $f(t) = (e^{-3t} - 5t^2)^2$
3. Find the Laplace transform of  $f(t) = te^{-4t} \cos 5t$
4. Find the Laplace transform of  $f(t) = (t^2 - 5t)u(t-3)$

Part II. (18 pts) Do three (3) of the following four problems:

5. Find the following inverse Laplace transform  $\mathcal{L}^{-1} \left( \left( \frac{5}{s^2} - \frac{1}{s^3} \right)^2 (s+1) \right)$
6. Find the following inverse Laplace transform  $\mathcal{L}^{-1} \left( \frac{5s-9}{s^2+2} \right)$
7. Find the following inverse Laplace transform  $\mathcal{L}^{-1} \left( \frac{3s+13}{s^2+8s+52} \right)$
8. Find the following inverse Laplace transform  $\mathcal{L}^{-1} \left\{ \frac{e^{-5s}}{s^2(s+2)} \right\}$

Part III. (65 pts) Do five (5) of the following six problems:

9. Use the method of Laplace transforms to solve the following linear differential equation
$$y' + 3y = 5t$$
$$y(0) = -2$$
10. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 3y' - 10y = 0$$
$$y(0) = 2, y'(0) = -1$$
11. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 10y' + 25y = t$$
$$y(0) = 0, y'(0) = -3$$
12. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 6y' + 8y = u(t - 2)$$
$$y(0) = 0, y'(0) = -1$$
13. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' + 16y = \cos 3t$$
$$y(0) = 0, y'(0) = -5$$
14. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 2y' + y = \delta(t - \pi)$$
$$y(0) = 1, y'(0) = -1$$