Name _____   Score _____

Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>** Attach this sheet to the front of your blue book/answer sheets.

Part I. Do 14 of the following 16 problems

1. (14 pts) Solve the initial value problem $x^2 \dfrac{dy}{dx} = y^2 - xy^2$, $y(2) = 1$

2. (14 pts) Find the general solution of the differential equation $x^2 y' + x(x+2) y = e^{-3x}$

3. (14 pts) Find the general solution of the differential equation
$$(3x^2 y + e^y - 4)dx + (x^3 + xe^y - 2y)dy = 0$$

4. (14 pts) Solve the initial value problem $(2x^2 + y^2)\dfrac{dy}{dx} = xy$, $y(1) = 1$

5. (14 pts) Use the method of *Reduction of Order* to find a second linearly independent solution y_2 of the differential equation $4x^2 y'' + 8xy' + y = 0$ given a known solution of $y_1 = 1/\sqrt{x}$.

6. (14 pts) Find the general solution of the differential equation $y'' + 6y' - 3y = 0$

7. (14 pts) Solve the initial value problem $y'' - 4y' + 13y = 0$, $y(0) = -2$, $y'(0) = 4$

8. (14 pts) Find the general solution of the differential equation $y'' + y' - 6y = 4 - 3x^2$

9. (14 pts) Find the general solution of the differential equation $y'' - 16y = 2e^{4x}$

10. (14 pts) Use the method of *Variation of Parameters* to find a particular solution of
$$y'' - 2y' + y = \frac{e^x}{1 + x^2}$$

11. (14 pts) Find the general solution of the differential equation $x^2 y'' + 5xy' + 4y = 0$

12. (14 pts) Solve the initial value problem $x^2 y'' - 6xy' + 10y = 0$, $y(2) = 12$, $y'(2) = -2$

13. (14 pts) A force of 2 lbs stretches a spring 1 foot. A mass which weighs 3.2 lbs is attached to the spring and the entire system is immersed in a medium that offers a damping force equal to 0.4 the instantaneous velocity. The mass is released at a point 1 ft above the equilibrium position with an initial downward velocity of 1 ft/s. Find the equation of motion.

14. (14 pts) Find the Laplace transform of $f(t) = \begin{cases} -3 & 0 \le t < 2 \\ 1 & 2 \le t < \infty \end{cases}$

15. (14 pts) Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 6y' + 8y = e^{-t}$$
$$y(0) = 2,\ y'(0) = 1$$

16. (14 pts) the method of Laplace transforms to solve the following linear differential equation
$$y'' - 4y' + 4y = te^{t}$$
$$y(0) = 1,\ y'(0) = -1$$

Part II. Do the following problem

17. (14 pts) Find two linearly independent power series solutions of $y'' - xy = 0$. (Find the first three non-zero terms of each power series.)