Math 3350 Competency Exam 26 January 2010
Version A-C

Part I. Calculus I competency.

1. (5 pts) Find the equation of the tangent line to the graph of the function
   \[ f(x) = \sqrt{2x^2 - 7} \] at \( x = 4 \).

2. (5 pts) Find the derivative of
   \[ a(x) = x^{5/3} - \frac{4}{x^2}. \]

3. (5 pts) Find the derivative of
   \[ b(x) = \frac{3x + 2}{4 - 5x}. \]

4. (5 pts) Find the derivative of
   \[ c(x) = e^{7x} \cos 6x. \]

5. (5 pts) Find the derivative of
   \[ e(x) = x^2 \ln x - 2x. \]

6. (5 pts) Find the derivative of
   \[ f(x) = (x^2 + x - 1)^7. \]

7. (5 pts) Find the indefinite integral
   \[ \int (10x^7 - 4\sqrt{x^3}) \, dx. \]

8. (5 pts) Find the definite integral of
   \[ \int_0^{\pi/2} (\cos x - \sin x) \, dx. \]

9. (5 pts) Find the indefinite integral
   \[ \int (3 - 4x)^7 \, dx. \]

10. (5 pts) Find the indefinite integral
    \[ \int \frac{1 - \sqrt{\ln x}}{x} \, dx. \]

Part II. Calculus II Competency.

11. (5 pts) Find the area the bounded in the first quadrant bounded between the graphs of
    \[ y = x^2 + 2x \] and \( y = 4x \).

12. (5 pts) Find the indefinite integral
    \[ \int x^2 e^x \, dx. \]

13. (5 pts) Find the partial fraction decomposition for the function
    \[ f(x) = \frac{3x - 1}{x^2 - 4x}. \]

14. (5 pts) Find the indefinite integral
    \[ \int \frac{2x - 1}{\sqrt{x} - x^2} \, dx. \]

15. (5 pts) Test the series for convergence:
    \[ \sum_{k=1}^{\infty} \frac{k}{2k^2 + 1}. \]

16. (5 pts) Test the series for convergence:
    \[ \sum_{k=1}^{\infty} k^2 \left( \frac{4}{7} \right)^{k+1}. \]

17. (5 pts) Find the first four non-zero terms of the MacLaurin series of
    \[ f(x) = \ln(1 - x^2) \]
Part III. Calculus III competency.

18. (5 pts) Find $f_x, f_y$ for $f(x, y) = xe^{x^2y}$

19. (5 pts) Find $f_{xx}, f_{yx}$ for $f(x, y) = xe^{x^2y}$

20. (5 pts) Find $f_x, f_y, f_z$ for $f(x, y, z) = \frac{x + \sqrt{y - z}}{z^3}$