

Answer the problems on **separate** paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. **Show all relevant steps** which lead to your solutions. Attach this question sheet to the front of your answer sheets.

1. (8 pts) Suppose that 20% of all children are left-handed. In a class of 12 children, what is the probability that at least three are left-handed?
2. (8 pts) A professor in a statistics course constructs a final exam by selecting seven questions at random from a list of twelve questions handed out in advance to the students. Suppose that a particular student has time to prepare answers to the first six of the twelve questions. What is the probability that the student will be prepared for at least five of the seven questions on the exam?
3. (8 pts) The number of raisins in a small cookie from a certain bakery has a Poisson distribution with  $\lambda = 2.2$ . Suppose we take (select independently) two of these cookies. What is the probability that each will contain at most two raisins?
4. (8 pts) The number of radioactive emissions (which are detected by a Geiger counter) from a sample of ore is (on the average) 34 per minute. Find the probability that during a 12-second span, there will be fewer than 7 emissions.
5. (8 pts) Suppose that 20% of all children are left-handed. Suppose a certain school has 10 classes of 17 children (each). If you check the classes one by one, what is the probability that the first left-handed child will be found in the third class?
6. A random variable has the density function
 
$$f(x) = \begin{cases} \frac{1}{21} \frac{1}{x^4} & \text{if } \frac{1}{4} < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
  - a) (4 pts) Verify that this is density function.
  - b) (8 pts) Find the mean and the variance for the distribution
7. Let  $x$  have a normal distribution with mean 26 and standard deviation 3.2. Find
  - a) (6 pts)  $P(23 < x < 28)$
  - b) (6 pts) the value  $t$  which is such that 2.5% of observations are larger than  $t$ .
8. (8 pts) The time for curing a latex sealant can be treated as a random variable having a normal distribution with mean  $\mu = 48$  minutes. Find its standard deviation, if the probability that the curing time will take longer than 55 minutes is 0.09.
9. (8 pts) A large construction firm has won 60% of the jobs for which it has bid. Suppose that this firm bids on 55 jobs over the next year. Use the normal approximation to the binomial to approximate the probability that it will win at most 28 of these jobs.
10. (8 pts) In a certain city, the daily usage of water (in million of gallons) can be treated as a random variable having an exponential distribution with  $\beta = 3.8$ . What is the probability that on any given day that the water supply will be able to provide between 6 and 12 million gallons?
11. A company manufacturers glass beakers whose fill capacity can be treated a random variable having a normal distribution with mean  $\mu = 0.5$  liters and variance  $\sigma^2 = 0.020$ .
  - a) (6 pts) If a random sample of size 26 is drawn, what is the probability that the sample mean will exceed 0.535?
  - b) (6 pts) How large a random sample must be drawn so that the probability that its mean will exceed 0.535 will be less than 0.05?