Key Exam II

1. Binomial $n = 16$ $p = 0.85$

$$P(x \geq 13) = 1 - B(12; 16, 0.85)$$
$$= 1 - 0.2101$$
$$= 0.7899$$

2. Hypergeometric $N = 22$ $(14+8)$, $n = 6$, $a = 8$

$$P(x \leq 2) = \sum_{k=0}^{2} h(k; 6, 8, 22)$$
$$= \frac{\binom{8}{0}\binom{14}{6}}{\binom{22}{6}} + \frac{\binom{8}{1}\binom{14}{5}}{\binom{22}{6}} + \frac{\binom{8}{2}\binom{14}{4}}{\binom{22}{6}} = 0.6305$$

3. Poisson Process $\alpha = 3/\text{week}$ $T = 2$ weeks

$\lambda = \alpha T = 6$ $P(x \leq 4) = F(4, 6) = 0.285$

4. Normal Distribution $X \rightarrow Z = \frac{X - \mu}{\sigma}$

a) $P(x < 70) = P\left(z < \frac{70-80}{\sqrt{90}}\right) = P(z < -1.054)$
$= F(-1.054) = 0.1460$

b) $P(x > 100) = P\left(z > \frac{100-80}{\sqrt{90}}\right) = P(z > 2.108)$
$= 1 - F(2.108) = 0.0175$

Need to interpolate $F(-1.054)$ and $F(2.108)$ from table

5. Normal Approximation to Binomial

$n = 125$ $\quad np = 85$ $\quad nq = 40$ ✓

$p = 0.68$

$\mu = np = 85$

$s = \sqrt{npq} = 5.215$

$P(X \leq 70)$ ← continuity correction

$\approx P\left(z \leq \frac{70.5 - 85}{5.215}\right)$

$= P(z \leq -2.780) = F(-2.780) = 0.0027$

6. Sampling Distribution (Large Sample) $\quad \bar{X} \rightarrow Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$P(0.3495 < \bar{X} < 0.3508)$

$= P\left(\frac{0.3495 - 0.3500}{0.0025/\sqrt{40}} < Z < \frac{0.3508 - 0.3500}{0.0025/\sqrt{40}}\right)$

$= P(-1.265 < Z < 2.024) = F(2.024) - F(-1.265) = 0.8756$

Need to interpolate $F(-1.265)$ and $F(2.024)$ from table

7. Maximum Error of Point Estimate

$\bar{X} = 3.667$

$s = 2.582$

$1 - \alpha = 98\%$

t-distribution with 5 deg. freedom

$t_{\alpha/2} = t_{0.01} = 3.365$

max error $\quad E = t_{\alpha/2} \frac{s}{\sqrt{n}} = 3.365 \frac{2.582}{\sqrt{6}} = 3.547$

8. Confidence Interval

$\bar{x} = \frac{57.7}{13} = 4.438$

$s^2 = \frac{n\sum x^2 - (\sum x)^2}{n(n-1)} = 0.311$

$s = \sqrt{s^2} = 0.558$

t-distribution with 12 d.f.

$\alpha = 0.05$

$t_{\alpha/2} = t_{0.025} = 2.179$

$E = t_{\alpha/2}\frac{s}{\sqrt{n}} = 0.337$

$\bar{x} - E < \mu < \bar{x} + E$ with 95% confidence

$4.101 < \mu < 4.775$ with 95% confidence

9. Binomial $n = 6$ $p = 0.18$

$P(x \le 1) = b(0; 6; 0.18) + b(1; 6, 0.18)$

$= \binom{6}{0} 0.18^0 0.82^6 + \binom{6}{1} 0.18^1 0.82^5$

$= 0.304 + 0.400 = 0.704$

10. Normal Distribution $X \rightarrow Z = \frac{X-\mu}{\sigma}$

$\mu = 0.19$

$\sigma = ?$

$P(X \ge 0.1965) = 0.12$

$\Updownarrow$

$P\left(z \ge \frac{0.1965 - 0.1900}{\sigma}\right) = 0.12$

$\Updownarrow$

$z_{0.12} = 1.175$ $\quad P(z \ge z_{0.12}) = 0.12$

$\frac{0.0065}{\sigma} = 1.175 \Rightarrow \sigma = 0.0055$

Need to interpolate $z_{0.12}$ from table