Solutions to Suggested Problems

6.21 n = 6 μ = 20 ambulance service claim of mean service time

$\bar{x}$ = 2.3

s = 6.39

$$t = \frac{23 - 20}{6.39/\sqrt{6}} = 1.15$$

df = 5

$t < t_{0.1} = 1.476$

$\therefore$ Claim is reasonable.

6.22 n = 10 μ = 0.5000 value determines whether process is in control

$\bar{x}$ = 0.5060

s = 0.0040

$$t = \frac{0.5060 - 0.5000}{0.0040/\sqrt{10}} = 4.743$$

df = 5

$t > t_{0.001} = 3.250$

$\therefore$ Process is out of control

6.24 n = 10

$\sigma^2 = 42.5$

Find P (3.14 < s < 8.94). Let $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$.

$$\frac{9\,(3.14)^2}{42.5} = 2.088 \qquad \frac{9\,(8.94)^2}{42.5} = 16.92$$

Find P (2.088 < χ^2 < 16.92) = 1 - 0.05 - 0.01 = 0.94

7.4 n = 50

$\bar{x}$ = 11,795

s = 14,054

$\alpha = 0.05$

maximum error (E) estimate:

$$E = |\bar{x} - \mu| < z_{\frac{0.05}{2}} \frac{14{,}054}{\sqrt{50}} = 1.960 \frac{14{,}054}{\sqrt{50}} = 3{,}896 \text{ with 95\% confidence}$$

7.5 Confidence interval: ($\bar{x}$ - E, $\bar{x}$ + E) = (11,795 - 3,896, 11,795 + 3,896) with 95% confidence.

7.6 n = 40

$\bar{x}$ = 12.73

s = 2.06

a) $\alpha = 0.01$

maximum error (E) estimate:

$$E = |\bar{x} - \mu| < z_{\frac{0.01}{2}} \frac{2.06}{\sqrt{40}} = 2.576 \frac{2.06}{\sqrt{40}} = 0.84 \text{ with 99\% confidence}$$

b) $\alpha = 0.02$

$$E_2 = |\bar{x} - \mu| < z_{\frac{0.02}{2}} \frac{2.06}{\sqrt{40}} = 2.326 \frac{2.06}{\sqrt{40}} = 0.76 \text{ with 98\% confidence}$$

Confidence interval: ($\bar{x}$ - E_2 , $\bar{x}$ + E_2) = (12.73- 0.76, 12.73 +0.76) with 98% confidence.

7.7 E = 0.5 min.

$$E = |\bar{x} - \mu| < z_{\frac{\alpha}{2}} \frac{2.06}{\sqrt{40}} = 0.5 \qquad z_{\frac{\alpha}{2}} = \frac{0.5\sqrt{40}}{2.06} = 1.535$$

$\alpha/2 = 1 - 0.9376 = 0.0624$ $\qquad \alpha = 0.1248$

Confidence level = 1 - 0.1248 = 0.8752 that maximum error is 0.5 min.

7.9 $n = 80$
$\bar{x} = 472.36$
$s = 62.35$

$E = |\bar{x} - \mu| < z_{\frac{\alpha}{2}} \dfrac{62.35}{\sqrt{80}} = 10$ $\qquad z_{\frac{\alpha}{2}} = \dfrac{10\sqrt{80}}{62.35} = 1.435$

$\alpha/2 = 1 - 0.9243 = 0.0757$ $\qquad \alpha = 0.1514$

Confidence level = 1 - 0.1514 = 0.8486 that maximum error is 10.

7.12 $\sigma = 60$
$1 - \alpha = 0.9$
$E = 10$

$E = |\bar{x} - \mu| < z_{\frac{0.1}{2}} \dfrac{60}{\sqrt{n}} \leq 10$ $\qquad 1.645 \dfrac{60}{\sqrt{n}} \leq 10$

$\sqrt{n} \geq \dfrac{1.645\,(60)}{10}$ $\qquad n \geq 97.4$

7.24 $n = 8$

$E_1 = |\bar{x} - \mu| < t_{\frac{0.05}{2}} \dfrac{0.54}{\sqrt{8}} = 2.365 \dfrac{0.54}{\sqrt{8}} = 0.45$ with 98% confidence

$\bar{x} = 2.1$
$s = 0.54$

$1 - \alpha = 0.95$
df = 7

Confidence interval $(\bar{x} - E_1, \bar{x} + E_1) = (2.1 - 0.45, 2.1 + 0.45)$ with 95% confidence.

7.27 a) Erroneously rejecting the null hypothesis that the dam was safe would cause unneeded renovation and/or repairs.

b) Erroneously accepting the null hypothesis that the dam was safe would cause loss of property and/or life.

7.30 $\mu = 3.0000$ $H_0: \mu = 3.0000$

$\sigma = 0.0250$ $H_1: \mu \neq 3.0000$

a) n = 30

H_0 will be rejected if $|\bar{x} - \mu| > 0.0040$

$$|z| = \frac{|\bar{x}-\mu|}{\sigma/\sqrt{n}} = \frac{0.0040}{0.0250/\sqrt{30}} = 1.012$$

P (|z| > 1.012) = 2 P (z > 1.012) = 2 (1 - 0.8670) = 0.2660

b) n = 30

Find P (x < 2.9960) + P(x > 3.0040) if $\mu = 3.0050$ prevails

$$z_a = \frac{2.9960-3.0050}{0.0250/\sqrt{30}} = -1.972$$

$$z_b = \frac{3.0040-3.0050}{0.0250/\sqrt{30}} = -0.219$$

Find $P(z < z_a) + P(z > z_b) = 0.02433 + 1 - 0.4125 = 0.6118$

7.32 $\mu = 100$

$\sigma = 12$

$n = 40$

$\alpha = 0.01$

Find $\bar{x}_0$ so that $P(x > \bar{x}_0) = 0.01$. Let $z_a = \dfrac{\bar{x}_0 - \mu}{\sigma/\sqrt{n}} = \dfrac{\bar{x}_0 - 100}{12/\sqrt{40}}$.

$P(z > z_a) = 0.01$ is equivalent to $z_a > z_{0.01} = 2.326$. Hence

$$\bar{x}_0 > 100 + 2.326\,\frac{12}{\sqrt{40}} = 100 + 4.41.$$

7.39 $n = 45$ $\mu = 73.2$
$\bar{x} = 76.7$ $\sigma = 8.6$

$\alpha = 0.01$

✐ $H_0 : \quad \mu = 7.32$
$H_1 : \quad \mu > 73.2$

✎ $\alpha = 0.01$

✓ $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Critical value: $z_\alpha = 2.326$

✔ $\dfrac{76.7 - 73.2}{8.6/\sqrt{45}} = 2.730$

✗ Reject H_0. The data provides sufficient evidence that mean aptitude score is more than 73.2.

7.40 $n = 35$ $\mu = 1.3$
$\bar{x} = 1.4707$
$s = 0.5235$

$\alpha = 0.05$

✐ $H_0 : \quad \mu = 1.3$
$H_1 : \quad \mu > 1.3$

✎ $\alpha = 0.05$

✓ $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Critical value: $z_\alpha = 1.645$

✔ $\dfrac{1.4707 - 1.3}{0.5325/\sqrt{35}} = 1.929$

✗ Reject H_0. The data provides sufficient evidence that mean cost is more than 1.3 (thousands).

7.41 $n = 64$ $\mu = 1{,}000$

$\bar{x} = 1{,}038$

$s = 146$

$\alpha = 0.05$

✎ $H_0: \quad \mu = 1000$

$H_1: \quad \mu > 1000$

✐ $\alpha = 0.05$

✓ $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Critical value: $z_\alpha = 1.645$

✔ $\dfrac{1{,}038 - 1{,}000}{146/\sqrt{64}} = 2.08$

✗ Reject H_0. The data provides sufficient evidence that mean number of acceptable pieces is more than 1,000.

7.43 $n = 60$ $\mu = 32.6$

$\bar{x} = 33.8$

$s = 6.1$

$\alpha = 0.05$

✎ $H_0: \quad \mu = 32.6$

$H_1: \quad \mu > 32.6$

✐ $\alpha = 0.05$

✓ $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ Critical value: $z_\alpha = 1.645$

✔ $\dfrac{33.8 - 32.6}{6.1/\sqrt{60}} = 1.52$

✗ Fail to reject H_0. The data does not provide sufficient evidence that mean travel time is more than 32.6 minutes.

7.44 n = 6 μ = 58,000
$\bar{x}$ = 58,392
s = 648

α = 0.05

✎ H_0 : μ = 58,000
H_1 : $\mu \neq$ 58,000

✎ α = 0.05

✓ $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Critical value: $t_{\alpha/2}$ = 2.571
df = 5

✔ $\dfrac{58{,}392 - 58{,}000}{648/\sqrt{5}} = 1.481$

✕ Fail to reject H_0. The data does not provide sufficient evidence that mean compressive strength is not than 58,000 psi.

7.48 n = 5 μ = 14.0
$\bar{x}$ = 14.4
s = 0.158

α = 0.05

✎ H_0 : μ = 14.0
H_1 : μ > 14.0

✎ α = 0.05

✓ $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Critical value: t_{α} = 2.132
df = 4

✔ $\dfrac{14.4 - 14.0}{0.158/\sqrt{4}} = 5.660$

✕ Reject H_0. The data provides sufficient evidence that mean tar content is more than 14.0.

7.49 $n = 5$ $\mu = 14.0$
$\bar{x} = 14.7$
$s = 0.742$

$\alpha = 0.05$

✐ $H_0: \quad \mu = 14.0$
$H_1: \quad \mu > 14.0$

✎ $\alpha = 0.05$

✓ $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ Critical value: $t_\alpha = 2.132$
df = 4

✔ $\dfrac{14.7 - 14.0}{0.742/\sqrt{4}} = 2.109$

✕ Fail to reject H_0. The data does not provide sufficient evidence that mean tar content is more than 14.0.

The increase in the standard deviation allows for more variability in the population data.

7.63 $\mu_1 = 0.249$ $\mu_2 = 0.255$
$\sigma_1 = 0.003$ $\sigma_2 = 0.002$

$$\mu_{x_1 - x_2} = \mu_1 - \mu_2 = -0.006$$

$$\sigma_{x_1 - x_2} = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.003605$$

$$P(x_1 - x_2 < 0) = P\left(z < \frac{0 - (-0.006)}{0.003605}\right) = P(z < 1.664) = 0.9519$$

7.64 $n_1 = 71$ $n_2 = 75$

$\bar{x}_1 = 83.2$ $\bar{x}_2 = 90.8$

$s_1 = 19.3$ $s_2 = 21.4$

$\alpha = 0.05$

(a)

✐ $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

✒ $\alpha = 0.05$

✓ $$z = \frac{(\bar{x}_1 - \bar{x}_2) - (0)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$ Critical value: $-z_{\alpha/2} = -1.960$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{19.3^2}{71} + \frac{21.4^2}{75}} = 3.369$$

✔ $$\frac{(83.2 - 90.8) - 0}{3.369} = -2.2558$$

✗ Reject H_0. The data provides sufficient evidence that the two population means are different.

(b)

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{19.3^2 + 21.4^2} = 28.81$$

$$d = \frac{|\mu_1 - \mu_2|}{\sigma} = \frac{|0 - (-12)|}{28.81} = 0.416$$

From Table 8 (c) the probability of failing to reject the null hypothesis when $\mu_1 - \mu_2 = -12$ prevails is 0.

7.65 $n_1 = 60$ $n_2 = 60$

$\bar{x}_1 = 292.5$ $\bar{x}_2 = 266.1$

$s_1 = 15.6$ $s_2 = 18.2$

$\alpha = 0.01$

✎ $H_0: \mu_1 - \mu_2 = 20$

$H_1: \mu_1 - \mu_2 > 20$

✒ $\alpha = 0.01$

✓ $$z = \frac{(\bar{x}_1 - \bar{x}_2) - (20)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

Critical value: $z_{\alpha} = 2.326$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{15.6^2}{71} + \frac{18.2^2}{75}} = 3.095$$

✔ $$\frac{(292.5 - 266.1) - 20}{3.095} = 2.068$$

✗ Fail to reject H_0. The data does not provide sufficient evidence that the mean weekly salaries for men are $20 more than the mean weekly salaries for women.

7.67 $n_1 = 8$ $n_2 = 10$

$\bar{x}_1 = 9.67$ $\bar{x}_2 = 7.43$

$s_1 = 1.81$ $s_2 = 1.48$

$\sigma_1 = \sigma_2 = \sigma$

$\alpha = 0.05$

✐ $H_0: \mu_1 - \mu_2 = 1.5$

$H_1: \mu_1 - \mu_2 > 1.5$

✍ $\alpha = 0.05$

✓ $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (1.5)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Critical value: $t_\alpha = 1.746$

df = 16

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{7(1.81)^2 + 9(1.48)^2}{16} = 2.665$$

✔ $$\frac{(9.67 - 7.43) - 1.5}{1.633\sqrt{\frac{1}{8} + \frac{1}{10}}} = 0.9556$$

✕ Fail to reject H_0. The data does not provide sufficient evidence that the mean deniers from the two spinning machines differ by more than 1.5.

7.68 $n_1 = 10$ $n_2 = 10$

$\bar{x}_1 = 70$ $\bar{x}_2 = 74$

$s_1 = 3.37$ $s_2 = 5.40$

$\sigma_1 = \sigma_2 = \sigma$

$\alpha = 0.05$

✐ $H_0 : \quad \mu_1 = \mu_2$

$H_1 : \quad \mu_1 < \mu_2$

✎ $\alpha = 0.05$

✓ $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (0)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Critical value: $-t_{\alpha} = -1.734$

df = 18

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2} = \frac{9(3.37)^2 + 9(5.40)^2}{18} = 20.258$$

✔ $$\frac{(70 - 74) - 0}{4.50\sqrt{\frac{1}{10} + \frac{1}{10}}} = -1.9876$$

✕ Reject H_0. The data provides sufficient evidence that the training under Method B is more effective.

7.69 $n_1 = 9$ $n_2 = 6$

$\bar{x}_1 = 58$ $\bar{x}_2 = 51.8$

$s_1 = 10.4$ $s_2 = 12.7$

$\sigma_1 = \sigma_2 = \sigma$

$\alpha = 0.01$

✎ $H_0: \quad \mu_1 = \mu_2$

$H_1: \quad \mu_1 \neq \mu_2$

✎ $\alpha = 0.01$

✓ $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (1.5)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Critical value: $t_{\alpha/2} = 3.012$

df = 13

$$s_p^2 = \frac{(n_1 - 1)\,s_1^2 + (n_2 - 1)\,s_2^2}{n_1 + n_2 - 2} = \frac{8(10.4)^2 + 5(12.7)^2}{13} = 128.594$$

✔ $$\frac{(58 - 51.8) - 0}{11.34\sqrt{\frac{1}{9} + \frac{1}{6}}} = 1.037$$

✗ Fail to reject H_0. The data does not provide sufficient evidence that the means from the two populations are different.

7.71 n = 10
$\bar{d}$ = -0.02
s_d = 0.0287

α = 0.05

✐ H_0 : $\bar{d} = 0$
H_1 : $\bar{d} \neq 0$

✎ α = 0.05

✓ $t = \frac{\bar{d} - 0}{s_d / \sqrt{n}}$ Critical value: $-t_{\alpha/2} = -2.262$

df = 9

✔ $\frac{-0.02 - 0}{0.0287/\sqrt{10}} = -2.2059$

✕ Fail to reject H_0. The data does not provide sufficient evidence that the two scales weigh differently.