Math 3322-001 Exam III-B October 17, 2007

Answer the problems on **separate** paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show **all relevant steps** which lead to your solutions. Attach this question sheet to the front of your answer sheets.

1. (18 pts) For each of the following geometric series, determine:

   i. Does the series converge?
   ii. If the series is convergent, then find its sum.

   a. $\sum_{n=1}^{\infty} \frac{4^n}{3^{n+1}} = \frac{4}{9} + \frac{16}{27} + \frac{64}{81} + \cdots$ b. $\sum_{n=1}^{\infty} 3\frac{2^{n+1}}{5^n} = \frac{12}{5} + \frac{24}{25} + \frac{48}{125} + \cdots$

2. (30 pts) Determine whether the following series converge or diverge. Clearly identify which method you are employing, how you are applying your method and what your conclusions are.

   a. $\sum_{n=1}^{\infty} \frac{1}{n^3 + 2n}$ b. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ c. $\sum_{n=1}^{\infty} 3n\left(\frac{4}{5}\right)^n$

3. (12 pts) Find the first 3 non-zero terms of the MacLaurin series for $f(x) = \sqrt{1+x}$.

4. (24 pts) Find the first 4 non-zero terms of the MacLaurin series for:

   a. $f(x) = \cosh x = \frac{e^x + e^{-x}}{2}$ b. $f(x) = \frac{1 - \cos x}{x}$

5. (18 pts) Using a MacLaurin series expansion for $\sin x$,

   a. approximate the value of $\sin 0.3$ by using the first 3 terms of the expansion

   b. find the maximum error which ensues by approximating the value of $\sin 0.3$ by the value obtained in a.

   c. from the maximum error in b., determine the accuracy of the approximation in a., i.e., determine how many decimal places are correct