Math 3322-001 | Exam III-B Make-up | October 17, 2007

Answer the problems on **separate** paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Do your own work. Show **all relevant steps** which lead to your solutions. Attach this question sheet to the <u>front</u> of your answer sheets.

1. (18 pts) For each of the following geometric series, determine:

   i. Does the series converge?
   ii. If the series is convergent, then find its sum.

   a. $\sum_{n=1}^{\infty} 36\frac{3^n}{4^{n+1}} = \frac{27}{4} + \frac{81}{16} + \frac{243}{64} + \cdots$ b. $\sum_{n=1}^{\infty} \frac{1}{12}\frac{4^{n-1}}{3^n} = \frac{1}{36} + \frac{1}{27} + \frac{4}{81} + \cdots$

2. (30 pts) Determine whether the following series converge or diverge. Clearly identify which method you are employing, how you are applying your method and what your conclusions are.

   a. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 24n}$ b. $\sum_{n=1}^{\infty} \frac{n!}{n^3 6^n}$ c. $\sum_{n=1}^{\infty} 3(n+2)\left(\frac{2}{3}\right)^n$

3. (12 pts) Find the first 3 non-zero terms of the MacLaurin series for $f(x) = \sqrt{9+x}$.

4. (24 pts) Find the first 4 non-zero terms of the MacLaurin series for:

   a. $f(x) = \frac{\sin x}{x} - \cos x$ b. $f(x) = (1 - e^{-x})(1 + e^{-x})$

5. (18 pts) Using a MacLaurin series expansion for $\cos x$,

   a1. approximate the value of $\cos 0.3$ by using the first 3 terms of the expansion
   a2. approximate the value of $\cos 0.8$ by using the first 3 terms of the expansion

   b1. find the maximum error which ensues by approximating the value of $\cos 0.3$ by the value obtained in a1.
   b2. find the maximum error which ensues by approximating the value of $\cos 0.8$ by the value obtained in a2.

   c1. from the maximum error in b1., determine the accuracy of the approximation in a1., i.e., determine how many decimal places are correct
   c2. from the maximum error in b2., determine the accuracy of the approximation in a2., i.e., determine how many decimal places are correct