Name ______________________ Key Exam III-A Score ______________

Section I. Short Answer Problems. Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

1. (4 pts) Consider the vector space $\mathbb{R}^3$ with basis $B = \{(-1,2,1)^T, (2,3,0)^T, (2,-1,-1)^T\}$. Suppose that $[x]_B = (-2,3,-1)^T$. Find the standard coordinates of x.

$\vec{x} = (6,6,-1)^T$

2. (8 pts) Consider the vector space $\mathbb{R}^3$ with basis $B = \{(0,1,-1)^T, (0,1,1)^T, (1,0,0)^T\}$. Suppose that $x = (-2,3,-1)^T$. Find $[x]_B$.

$[\vec{x}]_B = (2,1,-2)^T$

3. (10.5 pts) Consider the vector space $\mathbb{R}^4$. Consider the vectors $u = (-1,2,0,-1)^T$, $v = (2,1,-1,2)^T$, $w = (-1,1,-1,0)^T$. Find

$$A. \|u\|, \quad B. \|v+w\|, \quad C.\, d(u,v), \quad D. \cos\theta,$$

$\sqrt{6}$ $\sqrt{13}$ $\sqrt{20}$ 0

where θ is the angle between v and w. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. u and v, b. v and w, c. u-v and w

N $\perp$ N

4. (10.5 pts) Consider the inner product space $\mathbb{R}^4$ with inner product $< u,v >= u_1v_1 + 2u_2v_2 + 4u_3v_3 + u_4v_4$ for $u = (u_1,u_2,u_3,u_4)^T$ and $v = (v_1,v_2,v_3,v_4)^T$. Consider the vectors . Find $u = (-1,2,0,-1)^T$, $v = (2,1,-1,2)^T$, $w = (-1,1,-1,0)^T$

$$A. \|u\|, \quad B. \|v+w\|, \quad C.\, d(u,v), \quad D. \cos\theta,$$

$\sqrt{10}$ $\sqrt{29}$ $\sqrt{24}$ $\dfrac{4}{\sqrt{14}\sqrt{7}}$

where θ is the angle between v and w. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. u and v, b. v and w, c. u-v and w

$\perp$ N N

Skip solution for problems 5-7

Name ____________________ Exam III-A Score ________________

Section II. Take Home Exam - Take this sheet with you. Short Answer Problems. Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. *Do your own work.* **Show all relevant supporting steps!** Staple this sheet to the front of your answers. Due, 29 November, 5:00 pm.

1. (6 pts) Consider the vector space $\mathbb{R}^2$ with bases $B = \{(1,-1)^T, (1,1)^T\}$ and $B' = \{(1,2)^T, (2,1)^T\}$. Find the transition matrix from B to B'.

$\begin{bmatrix} -1 & \frac{1}{3} \\ 1 & \frac{1}{3} \end{bmatrix}$

2. (7 pts) Consider the inner product space P_1 with inner product $<f,g> = \int_0^1 f(x)g(x)dx$ for $f,g \in P_1$. Consider the vectors $u = 1-x, v = 2-6x, w = -1+3x$. Find

$A. \|u\|$, $B. \|v+w\|$, $C. d(u,v)$, $D. \cos\theta$,

$\sqrt{\frac{1}{3}}$ 1 $\sqrt{\frac{13}{3}}$ $\frac{-2}{\sqrt{4}\sqrt{1}} = -1$

where θ is the angle between v and w. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. u and v, b. v and w, c. u-v and w

$\perp$ $\parallel$ N

3. (6 pts) Consider the inner product space $\mathbb{R}^4$ with standard inner product (dot product). Consider the subspace S of $\mathbb{R}^4$ which is the span of the linearly independent vectors $u = (-1,0,-2,1)^T$, $v = (1,-1,1,1)^T$, $w = (1,-2,1,0)^T$

Use the Gram-Schmidt Orthogonalization method to find an orthogonal basis for S.

$\{(-1,0,-2,1)^T, \frac{1}{3}(2,-3,1,4)^T, \frac{1}{10}(-1,-11,-3,-7)^T\}$

4. (8 pts) Consider the matrix $A = \begin{bmatrix} 1 & -1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 0 & -1 \\ 1 & 1 & 0 & 1 & -1 \end{bmatrix}$. Find bases for each of the four fundamental subspaces of the matrix A.

basis $R(A) = \{(1,-1,1)^T, (-1,1,1)^T, (2,2,0)^T\}$

basis $R(A^T) = \{(1,0,0,\frac{1}{2},0)^T, (0,1,0,\frac{1}{2},-1)^T, (0,0,1,0,0)^T\}$

basis $N(A) = \{(-\frac{1}{2},-\frac{1}{2},0,1,0)^T, (0,1,0,0,1)^T\}$

basis $N(A^T)$ does not exist b/c $N(A^T) = \{(0,0,0)^T\}$

5. (6 pts) Find the least squares solution of the system $Ax = b$ where $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -1 & 3 \\ 1 & 1 \end{bmatrix}$ and

$b = \begin{bmatrix} -4 \\ 2 \\ 3 \\ -8 \end{bmatrix}$

$\bar{x} = (3/13, 7/13)^T$

6. (6 pts) Find the least squares regression quadratic polynomial for the data points: $(-2,0)$, $(0,3)$, $(1,2)$, $(0,-1)$, $(2,5)$, $(3,4)$

$p(x) \cong 1.340 + 0.928x + 0.0693x^2$

7. (6 pts) Consider the inner product space $C[0,4]$ with inner product $< f,g >= \int_0^4 f(x)g(x)dx$ for $f, g \in C[0,4]$. Construct an orthonormal basis for the subspace P_2 and then use it to construct a least squares approximation for the function $f(x) = \sqrt{x}$.

ON Basis $= \left\{ \frac{1}{2}, \frac{\sqrt{3}}{4}(x-2), \frac{3\sqrt{5}}{16}\left(x^2 - 4x + \frac{8}{3}\right) \right\}$

$\sqrt{x} \cong \frac{12}{35} + \frac{24}{35}x - \frac{1}{14}x^2$