Name ____________________________ Exam II-A Key Score ____________________

Section I. Short Answer Problems. Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

1. (5 pts) Find the determinant of the matrix A using an expansion by co-factors where

$$A = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 3 & -1 \\ 4 & -3 & 2 \end{bmatrix}$$

$|A| = 3$

2. (5 pts) Find the values of x for which the determinant of the matrix A is zero where

$$A = \begin{bmatrix} -1 & x & 0 \\ 1 & 0 & x \\ -x & 2 & 1 \end{bmatrix}$$

$\{-1, 0, 1\}$

3. (5 pts) Find the determinant of the matrix A by using elementary row operations to reduce A to a triangular matrix

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

$|A| = -1$

4. (5 pts) Give an example to show that the determinant of a sum of matrices is, in general, not equal to the sum of the determinants of the matrices.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

5. (5 pts) Determine whether the matrix A is singular where

$$A = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 0 & -1 \\ -2 & 1 & 3 \end{bmatrix}.$$

Non-singular

6. (6 pts) Let $A = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 2 \\ 1 & -2 & -1 \end{bmatrix}$. Find the determinant of: (a) A^T (b) A^4 (c) 5A

-2 16 -250

7. (6 pts) Let **u** = (-1,2,-4,1), **v** = (2,-1,-2,1), **w** = (2,-1,-1,3). Find:

(a) 2**u** - 4**v** + 3**w** (b) **v** - 3**u** - **w**

(-4,5,-3,7) (3,-6,11,-5)

Section II. Answer these problems on these test pages.

8. (3 pts) For each of the following sets answer the question: Is this set with its specified operations a vector space? If the answer is no, give a reason for why the answer is no.

A. $S = \left\{ \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$,

addition is standard matrix addition,
scalar multiplication is standard scalar multiplication

B. $S = \{(a,b) \mid a, b \in \mathbb{R}\}$,

addition is defined by $(a,b) \oplus (c,d) = (a+c, bd)$

scalar multiplication is defined by $\alpha(a,b) = (\alpha a, \alpha b)$

C. $S = \left\{ \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a^2 \end{bmatrix} \mid a \in \mathbb{R} \right\}$,

addition is standard matrix addition,
scalar multiplication is standard scalar multiplication

A. Yes No ________________________

B. Yes No No inverses

C. Yes No Not closed

9. (6 pts) For each of the following sets answer the question: Is this subset of P_3 a subspace of P_3? If the answer is no, give a reason for why the answer is no.

A. $\{ p(t) \mid p(-t) = p(t) \text{ for all } t \}$

B. $\{ p(t) \mid \int_0^4 p(t)dt = 0 \}$

C. $\{ p(t) \mid p(2) = 1 \}$

D. $\{ p(t) \mid p'(t) \text{ is constant} \}$

E. $\{ p(t) \mid p'(0) = 1 \}$

F. $\{ p(t) \mid p'(t) + 6\, p(t) = 0 \}$

A. Yes No ________________________

B. Yes No ________________________

C. Yes No Not closed under add.

D. Yes No ________________________

E. Yes No Not closed under add.

F. Yes No ________________________

10. (6 pts) For each of the following sets answer the question: Is this set a spanning set for the given vector space?

A. { (2,5), (-6,-15) } for $\mathbb{R}^2$ Yes No
B. { (1,-2), (-1,2), (-5,10) }, $\mathbb{R}^2$ Yes No
C. { (2,5,2), (-6,-15,6) }, $\mathbb{R}^3$ Yes No
D. { (-2,1,0), (2,-1,1), (2,1,3) }, $\mathbb{R}^3$ Yes No
E. $\{x, x^2 - x, x^2 + x\}$, P_2 Yes No
F. $\{x+1, x-1, x^2 + x\}$, P_2 Yes No

11. (6 pts) For each of the following sets answer the question: Is this set a linearly independent subset of the given vector space?

A. { (2,5), (-6,-15) } for $\mathbb{R}^2$ Yes No
B. { (1,-2), (-1,2), (-5,10) }, $\mathbb{R}^2$ Yes No
C. { (2,5,2), (-6,-15,6) }, $\mathbb{R}^3$ Yes No
D. { (-2,1,0), (2,-1,1), (2,1,3) }, $\mathbb{R}^3$ Yes No
E. $\{x, x^2 - x, x^2 + x\}$, P_2 Yes No
F. $\{x+1, x-1, x^2 + x\}$, P_2 Yes No

12. (6 pts) For each of the following sets answer the question: Is this set a basis for the given vector space? If the answer is no, give a reason for why the answer is no.

A. { (2,5), (-6,-15) } for $\mathbb{R}^2$
B. { (1,-2), (-1,2), (-5,10) }, $\mathbb{R}^2$
C. { (2,5,2), (-6,-15,6) }, $\mathbb{R}^3$
D. { (-2,1,0), (2,-1,1), (2,1,3) }, $\mathbb{R}^3$
E. $\{x, x^2 - x, x^2 + x\}$, P_2
F. $\{x+1, x-1, x^2 + x\}$, P_2

A. Yes No Not lin. indep.
B. Yes No Not lin. indep.
C. Yes No Not spanning set
D. Yes No
E. Yes No Not lin. indep.
F. Yes No

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Section 3. Take Home Exam - Take this sheet with you. Short Answer Problems. Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Staple this sheet to the front of your answers.

13. (6 pts) Find the least squares regression line for the set of points:

$\{(0,6), (4,3), (5,0), (8,4), (10,2), (8,6)\}$

$y \cong 4.445 + (-0.162)x$

14. (6 pts) Find the determinant of the matrix A where $A = \begin{bmatrix} -2 & 1 & 0 & 1 & -1 \\ -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & -1 & 2 \\ -1 & 0 & -1 & -1 & 0 \\ -2 & 2 & 2 & 0 & -1 \end{bmatrix}$

$|A| = -16$

15. (6 pts) Use Cramer's Rule to solve the system of equations $\begin{cases} 4x_1 - x_2 - x_3 & = & 1 \\ 2x_1 + 2x_2 + 3x_3 & = & 10 \\ 6x_1 - 2x_2 - 2x_3 & = & -1 \end{cases}$

$x_1 = \frac{3}{2} \quad x_2 = \frac{16}{2} \quad x_3 = \frac{-6}{2}$

16. (6 pts) Consider the matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix}$.

Find a basis for the row space of A. $\{(1,0,\frac{1}{2}), (0,1,-\frac{1}{2})\}$
Find a basis for the column space of A. $\{(1,4)^T, (-3,2)^T\}$
Find a basis for the null space of A. $\{(-\frac{1}{2}, \frac{1}{2}, 1)\}$

17. (9 pts) Consider the matrix $A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ -2 & -6 & 4 & -8 \end{bmatrix}$

Find a basis for the row space of A. $\{(1,0,1,-2), (0,1,-1,2)\}$
Find a basis for the column space of A. $\{(1,0,-2)^T, (3,1,-6)^T\}$
Find a basis for the null space of A. $\{(-1,1,1,0), (2,-2,0,1)\}$

18. (6 pts) Determine whether the vector b lies in the column space of the matrix A. If so, write b as a linear combination of the column vectors of A.

$$A = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 2 & -2 \\ -1 & 2 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ -2 \\ 7 \end{bmatrix}$$

$\bar{b} = 6\bar{a}_1 + 6\bar{a}_2 + 1a_3$