Section I. Short Answer Problems. Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** **STAPLE** this sheet to the front of your answers.

1. (4 pts) Consider the vector space $\mathbb{R}^4$ with basis $B = \{(-1,0,2,1)^T, (2,-1,3,0)^T, (0,2,-1,-1)^T, (3,-1,0,2)^T\}$. Suppose that $[x]_B = (-2,4,1,-3)^T$. Find the standard coordinates of $x$.

2. (8 pts) Consider the vector space $\mathbb{R}^4$ with basis $B = \{(-1,0,1,-1)^T, (1,-1,0,1)^T, (-1,1,-1,0)^T, (0,1,-1,1)^T\}$. Suppose that $x = (-2,4,1,-3)^T$. Find $[x]_B$.

3. (10.5 pts) Consider the vector space $\mathbb{R}^5$. Consider the vectors $u = (-1,2,0,2,3)^T$, $v = (0,-2,1,-1,2)^T$, $w = (-1,1,-1,3,2)^T$. Find

$$A. \| u \|, \ B. \| v + w \|, \ C. d(u,v), \ D. \cos \theta,$$

where $\theta$ is the angle between $v$ and $w$. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. $u$ and $v$,  b. $v$ and $w$,  c. $u-v$ and $w$.

4. (10.5 pts) Consider the inner product space $\mathbb{R}^5$ with inner product $<u,v> = u_1v_1 + 3u_2v_2 + 4u_3v_3 + 2u_4v_4 + 4u_5v_5$ for $u = (u_1,u_2,u_3,u_4,u_5)^T$ and $v = (v_1,v_2,v_3,v_4,v_5)^T$. Consider the vectors $u = (-1,2,0,2,3)^T$, $v = (0,-2,1,-1,2)^T$, $w = (-1,1,-1,3,2)^T$. Find

$$A. \| u \|, \ B. \| v + w \|, \ C. d(u,v), \ D. \cos \theta,$$

where $\theta$ is the angle between $v$ and $w$. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. $u$ and $v$,  b. $v$ and $w$,  c. $u-v$ and $w$.
5. (10.5 pts) Consider the inner product space \( P_2 \) with (coefficient) inner product 
\[
\langle p, q \rangle = a_0b_0 + a_1b_1 + 2a_2b_2
\]
for 
\[
p(x) = a_0 + a_1x + a_2x^2 \quad \text{and} \quad q(x) = b_0 + b_1x + b_2x^2.
\]
Consider the vectors 
\[
u = 1 + 2x - 2x^2, \quad v = 2 - x^2, \quad w = -1 + x - x^2.
\]
Find 
\[
A. \| u \|, \quad B. \| v + w \|, \quad C. d(u, v), \quad D. \cos \theta,
\]
where \( \theta \) is the angle between \( v \) and \( w \). Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. \( u \) and \( v \),  
b. \( v \) and \( w \),  
c. \( u - v \) and \( w \)

6. (8 pts) Consider the inner product space \( \mathbb{R}^5 \) with inner product 
\[
\langle u, v \rangle = u_1v_1 + 3u_2v_2 + 4u_3v_3 + 2u_4v_4 + 4u_5v_5
\]
for 
\[
u = (u_1, u_2, u_3, u_4, u_5)^T \quad \text{and} \quad v = (v_1, v_2, v_3, v_4, v_5)^T.
\]
Consider the vectors 
\[
u = (-1, 2, 0, 2, 3)^T, \quad v = (0, -2, 1, -1, 2)^T, \quad w = (-1, 1, -1, 3, 2)^T.
\]
Find \( \text{proj}_u v \) and \( \text{proj}_v w \).

7. (10 pts) Consider the inner product space \( \mathbb{R}^3 \) with standard inner product (dot product). Let \( S \) be the subspace of \( \mathbb{R}^3 \) which is the span of the vectors 
\[
u = (1, 1, 0)^T \quad \text{and} \quad v = (1, 0, -1)^T.
\]
Find the projection of 
\[
\text{proj}_S w = (2, 3, 4)^T
\]
on to \( S \).