Name _____ Exam III-B Make-up Score _____

Section I. Short Answer Problems. Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>** ***<u>STAPLE</u>*** this sheet to the front of your answers.

1. (4 pts) Consider the vector space $\mathbb{R}^4$ with basis $B=\{(-1,0,2,1)^T,(2,-1,3,0)^T,(0,2,-1,-1)^T,(3,-1,0,2)^T\}$. Suppose that $[x]_B=(-2,4,1,-3)^T$. Find the standard coordinates of $x$.

2. (8 pts) Consider the vector space $\mathbb{R}^4$ with basis $B=\{(-1,0,1,-1)^T,(1,-1,0,1)^T,(-1,1,-1,0)^T,(0,1,-1,1)^T\}$. Suppose that $x=(-2,4,1,-3)^T$. Find $[x]_B$.

3. (10.5 pts) Consider the vector space $\mathbb{R}^5$. Consider the vectors $u=(-1,2,0,2,3)^T$, $v=(0,-2,1,-1,2)^T$, $w=(-1,1,-1,3,2)^T$. Find

$$A.\|u\|,\ B.\|v+w\|,\ C.\ d(u,v),\ D.\cos\theta,$$

where $\theta$ is the angle between *v* and *w*. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. *u* and *v*, b. *v* and *w*, c. *u*-*v* and *w*

4. (10.5 pts) Consider the inner product space $\mathbb{R}^5$ with inner product $<u,v>=u_1v_1+3u_2v_2+4u_3v_3+2u_4v_4+4u_5v_5$ for $u=(u_1,u_2,u_3,u_4,u_5)^T$ and $v=(v_1,v_2,v_3,v_4,v_5)^T$. Consider the vectors $u=(-1,2,0,2,3)^T$, $v=(0,-2,1,-1,2)^T$, $w=(-1,1,-1,3,2)^T$. Find

$$A.\|u\|,\ B.\|v+w\|,\ C.\ d(u,v),\ D.\cos\theta,$$

where $\theta$ is the angle between *v* and *w*. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. *u* and *v*, b. *v* and *w*, c. *u*-*v* and *w*

5. (10.5 pts) Consider the inner product space $P_2$ with (coefficient) inner product $< p,q >= a_0b_0 + a_1b_1 + 2a_2b_2$ for $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$. Consider the vectors $u = 1 + 2x - 2x^2$, $v = 2 - x^2$, $w = -1 + x - x^2$. Find

$$A. \|u\|,\ B. \|v+w\|,\ C.\ d(u,v),\ D. \cos\theta,$$

where $\theta$ is the angle between *v* and *w*. Also, determine whether any of the following pairs of vectors are orthogonal, parallel or neither:

a. *u* and *v*, b. *v* and *w*, c. *u*-*v* and *w*

6. (8 pts) Consider the inner product space $\mathbb{R}^5$ with inner product $< u,v >= u_1v_1 + 3u_2v_2 + 4u_3v_3 + 2u_4v_4 + 4u_5v_5$ for $u = (u_1, u_2, u_3, u_4, u_5)^T$ and $v = (v_1, v_2, v_3, v_4, v_5)^T$. Consider the vectors $u = (-1, 2, 0, 2, 3)^T$, $v = (0, -2, 1, -1, 2)^T$, $w = (-1, 1, -1, 3, 2)^T$. Find $\text{proj}_u\, v$ and $\text{proj}_v\, w$.

7. (10 pts) Consider the inner product space $\mathbb{R}^3$ with standard inner product (dot product). Let $S$ be the subspace of $\mathbb{R}^3$ which is the span of the vectors $u = (1,1,0)^T$ *and* $v = (1,0,-1)^T$. Find the projection of $w = (2,3,4)^T$ on to $S$.