Section I. Short Answer Problems. Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

1. (5 pts) Find the determinant of the matrix $A$ using an expansion by co-factors

   $$A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ -2 & 0 & 3 & 1 \\ -1 & -1 & 2 & 0 \\ 0 & 2 & -1 & 3 \end{bmatrix}$$

2. (5 pts) Find the values of $x$ for which the determinant of the matrix $A$ is zero

   $$A = \begin{bmatrix} -1 & x & 1 & 0 \\ 1 & 1 & x & -2 \\ x & 0 & 1 & -1 \\ 2 & -1 & 2 & 1 \end{bmatrix}$$

3. (5 pts) Find the determinant of the matrix $A$ by using elementary row operations to reduce $A$ to a triangular matrix $A = \begin{bmatrix} -1 & 2 & 1 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 2 & -1 & 2 \\ 1 & 0 & 2 & 2 \end{bmatrix}$

4. (5 pts) Give an example to show that the determinant of a sum of matrices is, in general, not equal to the sum of the determinants of the matrices.

   $$A = \begin{bmatrix} -1 & 2 & 0 & 1 \\ 3 & 2 & 1 & -1 \\ 0 & 2 & 1 & -3 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

5. (5 pts) Determine whether the matrix $A$ is singular where $A = \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & -2 & 1 & 3 \\ 1 & -2 & -1 & 1 \\ 2 & 0 & -3 & 1 \end{bmatrix}$

6. (6 pts) Let $A = \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & -2 & 1 & 3 \\ 1 & -2 & -1 & 1 \\ 2 & 0 & -3 & 1 \end{bmatrix}$. Find the determinant of: (a) $A^T$ (b) $A^5$ (c) $6A$

7. (6 pts) Let $u = (-1,2,-4,1)$, $v = (2,-1,-2,1)$, $w = (2,-1,-1,3)$. Find:

   (a) $-2u + 4v + 5w$  (b) $2v - 4u - 3w$
Section II. Answer these problems on these test pages.

8. (3 pts) For each of the following sets answer the question: Is this set with its specified operations a vector space? If the answer is no, give a reason for why the answer is no.

A. \( S = \{ p(x) \in P \mid p'(1) = p'(-1) \} \),
   addition is standard polynomial addition,
   scalar multiplication is standard scalar multiplication for polynomials

B. \( S = \{(a, b) \mid a, b \in \mathbb{R}\} \),
   addition is defined by \((a, b) \oplus (c, d) = (a + c, b - d)\)
   scalar multiplication is defined by \(\alpha(a, b) = (\alpha a, \alpha b)\)

C. \( S = \begin{bmatrix} a & 0 & -b \\ 0 & a - c & 0 \\ -b & 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{R} \),
   addition is standard matrix addition,
   scalar multiplication is standard scalar multiplication

A. Yes No ____________________________________________
B. Yes No ____________________________________________
C. Yes No ____________________________________________

9. (6 pts) For each of the following sets answer the question: Is this subset of \( P_3 \) a subspace of \( P_3 \)? If the answer is no, give a reason for why the answer is no.

A. \( \{ p(t) \in P_3 \mid p(t) \ast p'(t) = 0 \text{ for all } t \} \)
B. \( \{ p(t) \in P_3 \mid p''(t) = 0 \} \)
C. \( \{ p(t) \in P_3 \mid p(0) \ast p(1) = 0 \} \)
D. \( \{ p(t) \in P_3 \mid p(0) \ast p(1) = 1 \} \)
E. \( \{ p(t) \in P_3 \mid p'(2) = p'(4) \} \)
F. \( \{ p(t) \in P_3 \mid p(t) = t \ast p'(t) \} \)

A. Yes No ____________________________________________
B. Yes No ____________________________________________
C. Yes No ____________________________________________
D. Yes No ____________________________________________
E. Yes No ____________________________________________
F. Yes No ____________________________________________
10. (6 pts) For each of the following sets answer the question: Is this set a spanning set for the given vector space?

A. \{ (2,5), (-6,-15) \} for \( \mathbb{R}^2 \) Yes No
B. \{ (1,-2), (-1,2), (-2,1) \}, \( \mathbb{R}^2 \) Yes No
C. \{ (2,5,2), (-3,-5,2), (-3,2,1) \}, \( \mathbb{R}^3 \) Yes No
D. \{ (-2,3,4), (2,-1,-2), (2,5,4) \}, \( \mathbb{R}^3 \) Yes No
E. \{ x, x^2 - x, x^2 + x \}, \( P_2 \) Yes No
F. \{ x+1, x-1, x^2 + x^3 \}, \( P_2 \) Yes No

11. (6 pts) For each of the following sets answer the question: Is this set a linearly independent subset of the given vector space?

A. \{ (2,5), (-6,-15) \} for \( \mathbb{R}^2 \) Yes No
B. \{ (1,-2), (-1,2), (-2,1) \}, \( \mathbb{R}^2 \) Yes No
C. \{ (2,5,2), (-3,-5,2), (-3,2,1) \}, \( \mathbb{R}^3 \) Yes No
D. \{ (-2,3,4), (2,-1,-2), (2,5,4) \}, \( \mathbb{R}^3 \) Yes No
E. \{ x, x^2 - x, x^2 + x \}, \( P_2 \) Yes No
F. \{ x+1, x-1, x^2 + x^3 \}, \( P_2 \) Yes No

12. (6 pts) For each of the following sets answer the question: Is this set a basis for the given vector space? If the answer is no, give a reason for why the answer is no.

A. \{ (2,5), (-6,-15) \} for \( \mathbb{R}^2 \)
B. \{ (1,-2), (-1,2), (-2,1) \}, \( \mathbb{R}^2 \)
C. \{ (2,5,2), (-3,-5,2), (-3,2,1) \}, \( \mathbb{R}^3 \)
D. \{ (-2,3,4), (2,-1,-2), (2,5,4) \}, \( \mathbb{R}^3 \)
E. \{ x, x^2 - x, x^2 + x \}, \( P_2 \)
F. \{ x+1, x-1, x^2 + x^3 \}, \( P_2 \)

A. Yes No _______________________________
B. Yes No _______________________________
C. Yes No _______________________________
D. Yes No _______________________________
E. Yes No _______________________________
F. Yes No _______________________________