Name ___________________________ Exam II-B Makeup Score ____________________

Section I. Short Answer Problems. Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>** Attach this sheet to the front of your answers.

1. (5 pts) Find the determinant of the matrix A using an expansion by co-factors

where $A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ -2 & 0 & 3 & 1 \\ -1 & -1 & 2 & 0 \\ 0 & 2 & -1 & 3 \end{bmatrix}$

2. (5 pts) Find the values of x for which the determinant of the matrix A is zero

where $A = \begin{bmatrix} -1 & x & 1 & 0 \\ 1 & 1 & x & -2 \\ x & 0 & 1 & -1 \\ 2 & -1 & 2 & 1 \end{bmatrix}$

3. (5 pts) Find the determinant of the matrix A by using elementary row operations to reduce A to a triangular matrix $A = \begin{bmatrix} -1 & 2 & 1 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 2 & -1 & 2 \\ 1 & 0 & 2 & 2 \end{bmatrix}$

4. (5 pts) Give an example to show that the determinant of a sum of matrices is, in general, not equal to the sum of the determinants of the matrices.

5. (5 pts) Determine whether the matrix A is singular where $A = \begin{bmatrix} -1 & 2 & 0 & 1 \\ 3 & 2 & 1 & -1 \\ 0 & 2 & 1 & -3 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

6. (6 pts) Let $A = \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & -2 & 1 & 3 \\ 1 & -2 & -1 & 1 \\ 2 & 0 & -3 & 1 \end{bmatrix}$. Find the determinant of: (a) A^T (b) A^5 (c) 6A

7. (6 pts) Let **u** = (-1,2,-4,1), **v** = (2,-1,-2,1), **w** = (2,-1,-1,3). Find:

(a) -2**u** + 4**v** + 5**w** (b) **2v** - 4**u** - **3w**

Section II. Answer these problems on these test pages.

8. (3 pts) For each of the following sets answer the question: Is this set with its specified operations a vector space? If the answer is no, give a reason for why the answer is no.

A. S = $\{p(x) \in P \mid p'(1) = p'(-1)\}$,
addition is standard polynomial addition,
scalar multiplication is standard scalar multiplication for polynomials

B. S = $\{(a,b) \mid a,b \in \mathbb{R}\}$,
addition is defined by $(a,b) \oplus (c,d) = (a+c, b-d)$
scalar multiplication is defined by $\alpha(a,b) = (\alpha a, \alpha b)$

C. S = $\left\{ \begin{bmatrix} a & 0 & -b \\ 0 & a-c & 0 \\ -b & 0 & c \end{bmatrix} \mid a,b,c \in \mathbb{R} \right\}$,
addition is standard matrix addition,
scalar multiplication is standard scalar multiplication

A. Yes No ____
B. Yes No ____
C. Yes No ____

9. (6 pts) For each of the following sets answer the question: Is this subset of P_3 a subspace of P_3? If the answer is no, give a reason for why the answer is no.

A. { p(t) $\in P_3$ | p(t) * p'(t) = 0 for all t }

B. { p(t) $\in P_3$ | p''(t) = 0 }

C. { p(t) $\in P_3$ | p(0)*p(1) = 0 }

D. { p(t) $\in P_3$ | p(0)*p(1) = 1 }

E. { p(t) $\in P_3$ | p'(2) = p'(4) }

F. { p(t) $\in P_3$ | p(t) =t* p'(t) }

A. Yes No ____
B. Yes No ____
C. Yes No ____
D. Yes No ____
E. Yes No ____
F. Yes No ____

10. (6 pts) For each of the following sets answer the question: Is this set a spanning set for the given vector space?

A. { (2,5), (-6,-15) } for $\mathbb{R}^2$ Yes No

B. { (1,-2), (-1,2), (-2,1) }, $\mathbb{R}^2$ Yes No

C. { (2,5,2), (-3,-5,2), (-3,2,1) }, $\mathbb{R}^3$ Yes No

D. { (-2,3,4), (2,-1,-2), (2,5,4) }, $\mathbb{R}^3$ Yes No

E. $\{x, x^2-x, x^2+x\},\ P_2$ Yes No

F. $\{x+1, x-1, x^2+x\},\ P_2$ Yes No

11. (6 pts) For each of the following sets answer the question: Is this set a linearly independent subset of the given vector space?

A. { (2,5), (-6,-15) } for $\mathbb{R}^2$ Yes No

B. { (1,-2), (-1,2), (-2,1) }, $\mathbb{R}^2$ Yes No

C. { (2,5,2), (-3,-5,2), (-3,2,1) }, $\mathbb{R}^3$ Yes No

D. { (-2,3,4), (2,-1,-2), (2,5,4) }, $\mathbb{R}^3$ Yes No

E. $\{x, x^2-x, x^2+x\},\ P_2$ Yes No

F. $\{x+1, x-1, x^2+x\},\ P_2$ Yes No

12. (6 pts) For each of the following sets answer the question: Is this set a basis for the given vector space? If the answer is no, give a reason for why the answer is no.

A. { (2,5), (-6,-15) } for $\mathbb{R}^2$

B. { (1,-2), (-1,2), (-2,1) }, $\mathbb{R}^2$

C. { (2,5,2), (-3,-5,2), (-3,2,1) }, $\mathbb{R}^3$

D. { (-2,3,4), (2,-1,-2), (2,5,4) }, $\mathbb{R}^3$

E. $\{x, x^2-x, x^2+x\},\ P_2$

F. $\{x+1, x-1, x^2+x\},\ P_2$

A. Yes No ______________________________________

B. Yes No ______________________________________

C. Yes No ______________________________________

D. Yes No ______________________________________

E. Yes No ______________________________________

F. Yes No ______________________________________