MATH 2360-005 | Exam I
Makeup B | 20 September 2012

Name __________________________ Score ____________________

Answer the problems on separate paper.

DO NOT ANSWER MORE THAN TWO PROBLEMS PER SIDE PER SHEET OF PAPER.

You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

1. (4pts; 8pts) Solve the following systems of linear equations:

a. $$\begin{aligned} 3x - 5y &= -6 \\ -4x + 6y &= 2 \end{aligned}$$

b. $$\begin{aligned} 3x - 3y + 2z - 3w &= 6 \\ 6x - 12y + 8z - 2w &= -4 \end{aligned}$$

2. (6 pts) Identify which of the following matrices are

(i) in reduced row echelon form [RREF],
(ii) in row echelon form but not reduced row echelon form [ROW],
(iii) not in row echelon form [NOT].

Use the labels RREF, ROW and NOT to denote your answers. (Note options (i), (ii) and (iii) are mutually exclusive.)

a. $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ f. $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

3. (4pts; 8pts) Represent each of the following systems of equations via an augmented matrix. Then, find the solution set of the system of equations by applying Gaussian Elimination with back substitution to the representing augmented matrix or by applying Gauss-Jordan Elimination to the representing augmented matrix. Clearly annotate your steps as you apply Gaussian Elimination or Gauss-Jordan Elimination using the notation developed in the textbook and in the lecture.

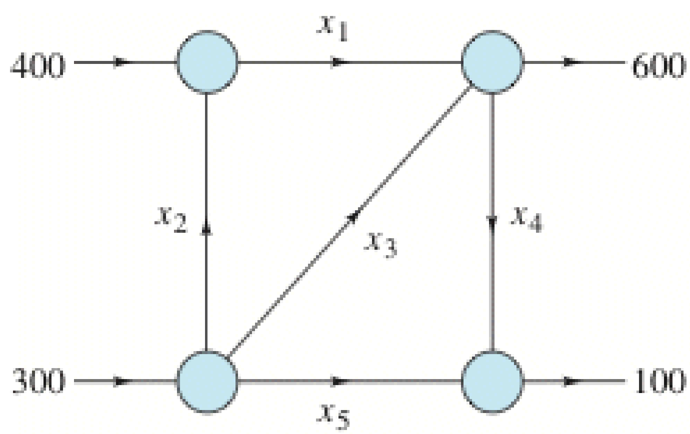
a. $$\begin{aligned} 3x + 3y &= 4 \\ 5x + 4y &= 7 \end{aligned}$$

b. $$\begin{aligned} 2x + 4y - 3z + 2w &= -1 \\ 5x + 8y - 6z + 4w &= -1 \end{aligned}$$

4. (12 pts) Determine the polynomial function of degree 2 whose graph passes through the given 3 points and then sketch the graph of the polynomial function, showing the given points:

(-2,10), (1,-14), (4,16)

5. (12 pts) The figure below shows the flow of traffic (in vehicles per hour) through a network of streets. Construct a linear system of equations to represent this network flow system. Suppose that additional measurements show that $x_3 = 125$ and $x_5 = 25$, then find the values of the other traffic flow variables for this system.



6. (8 pts) Consider the matrices

A = $\begin{bmatrix} 2 & -2 & -3 \\ -4 & -1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 4 & -1 \\ -3 & -5 \end{bmatrix}$ C = $\begin{bmatrix} -2 & -1 & 3 \\ -4 & 2 & -1 \end{bmatrix}$ D = $\begin{bmatrix} 2 & -3 \\ -6 & 2 \end{bmatrix}$

Perform each of the following operations, if possible. If it is not possible, state so.

a. 4C - 5A b. BA c. BD^T d. B^TC

7. (6 pts) Let $A = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$. Show that there exists a 2×2 matrix B such that AB $\neq$ BA.

8. (4pts; 10pts) Find the inverse of each of the following matrices, if the inverse exists. If the inverse does not exist, state so.

a. $\begin{bmatrix} 5 & -3 \\ 3 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 4 & 1 & -1 \end{bmatrix}$

9. (4pts; 6pts) For each of the following pairs of matrices find an elementary matrix E such that EA = B.

a. $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$

b. $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 1 & 2 \\ -2 & -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & -2 \\ 3 & 1 & 2 \\ -2 & -1 & -2 \end{bmatrix}$

10. (12 pts) Find an *LU* factorization for the following matrix:

$$\begin{bmatrix} 12 & -15 & -1 \\ -4 & 7 & 1 \\ 3 & -3 & 3 \end{bmatrix}$$