Math 2360 Exam III 18 November 2009
Form B

Section I For each of the problems in the this section at least one of the choices is correct. For some of the problems more than one of the choices is correct. Record your answers for problems in this section on the answer sheet (last page).

1. (9 pts) Determine whether the following are linear transformations from $\mathbb{R}^4$ to $\mathbb{R}^3$

a. $$L(\boldsymbol{x}) = \begin{bmatrix} \frac{x_1 + x_3}{2} + 2x_4 \\ x_1 - \frac{x_2 + x_4}{2} \\ 2x_2 - 4x_4 \end{bmatrix}$$

b. $$L(\boldsymbol{x}) = \begin{bmatrix} x_1 - 2x_2 + 4x_4 \\ x_1 + \frac{x_2 + x_3 + x_4}{x_1} \\ \frac{x_1 + x_2 + x_3}{x_4} - x_4 \end{bmatrix}$$

c. $$L(\boldsymbol{x}) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2. (9 pts) Determine whether the following are linear transformations from P_4 to P_4

a. $L(p(x)) = p(x) + xp'(x) + x^2 p''(x)$ b. $L(p(x)) = \dfrac{p(x) - p(-x)}{x}$

c. $L(p(x)) = p(0)p''(0) + p(x)$

Section II Continue the answers for these problems on the answer sheet. Append additional pages as needed. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

3. (10 pts) Consider the linear transformation mapping $\mathbb{R}^4$ to $\mathbb{R}^2$ given by

$$L(\boldsymbol{x}) = [x_1 + 3x_2 + 2x_3 + 4x_4, x_1 - 2x_2 + x_3 - 2x_4]^T$$

a. Find the kernel of L.
b. Find the dimension of the range of L.

4. (10 pts) Consider the linear transformation mapping P_4 to P_4 given by

$$L(p(x)) = \frac{1}{x}\int_0^x p(t)dt$$

a. Find the kernel of L.
b. Find the dimension of the range of L.

5. (8 pts) Consider the linear transformation mapping $\mathbb{R}^4$ to $\mathbb{R}^3$ given by

$$L(\boldsymbol{x}) = [x_1 - 2x_2 + 3x_3 - 4x_4, 2x_1 + 4x_2 - x_3 - 3x_4, 2x_2 + 2x_4]^T$$

Find the standard matrix representation for *L*.

6. (16 pts) Consider the linear transformation mapping $\mathbb{R}^3$ to $\mathbb{R}^2$ given by

$$L(\boldsymbol{x}) = [x_1 - 2x_2 - 3x_3, 2x_1 - 3x_2 + 2x_3]^T$$

Find a matrix *A* which represents *L* with respect the standard basis $[\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3]$ in $\mathbb{R}^3$ and the ordered basis $[\boldsymbol{b}_1, \boldsymbol{b}_2]$ in $\mathbb{R}^2$ where $b_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

7. (10 pts) Find the equation of the plane in $\mathbb{R}^3$ which is normal to the vector $\boldsymbol{N} = [1,\ 2,\ -3]^T$ and which passes through the point $\boldsymbol{P}_0 = [-3,\ 2,\ 2]^T$.

8. (10 pts) Find the distance in $\mathbb{R}^3$ from the point $\boldsymbol{P}_0 = [-1,\ -2,\ 3]^T$ to the plane given by $2x - 2y + z = -6$.

9. (10 pts) Let *S* be the subspace in $\mathbb{R}^4$ which is spanned by the set $\left\{[1,\ -2,\ 1, -1]^T,\ [-1,\ 2,\ 1, -1]^T\right\}$. Find a basis for $S^{\perp}$.

10. (10 pts) Find the least squares solution $\hat{\boldsymbol{x}}$ of the linear system $\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & -1 \\ 1 & 0 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix}$

Name _____ Form B

Answers

1.

i. Yes No ii. Yes No iii. Yes No

2.

i. Yes No ii. Yes No iii. Yes No