Math 2360 Exam II 28 October 2009

Make-up Form B

Section I

For each of the following problems at least one of the choices is correct. For many of the problems more than one of the choices is correct. Record your answers for each problem on the answer sheet (last page). For theis section, turn in (only) the answer sheet (with your answers on it) at the end of the exam. Do your own work. You may keep the exam for your own records.

1. (12 pts) Consider the V set of polynomials with degree less than 3. Define "addition" for V by if $p,q \in V$ with $p(x) = a + bx + cx^2$ and $q(x) = \alpha + \beta x + \gamma x^2$, then $p \oplus q$ is defined by $p \oplus q(x) = \sqrt[3]{a^3 + \alpha^3} + \sqrt[3]{b^3 + \beta^3}\,x + \sqrt[3]{c^3 + \gamma^3}\,x^2$. (We will use symbol $\oplus$ for "addition" since this is not the usual addition for polynomials.) Define scalar multiplication for V by if $p \in V$ with $p(x) = a + bx + cx^2$ *and* $\alpha \in \mathbb{R}$ then αp is defined by $\alpha p(x) = \alpha a + \alpha bx + \alpha cx^2$. (This is usual scalar multiplication for polynomials.)

Recall the vector space axioms for addition for a vector space *V*.

i. Addition is commutative, i.e. for every ***p, q*** in *V* we have $\boldsymbol{p} \oplus \boldsymbol{q} = \boldsymbol{q} \oplus \boldsymbol{p}$

ii. Addition is associative, i.e., for every ***p, q, r*** in V we have $(\boldsymbol{p} \oplus \boldsymbol{q}) \oplus \boldsymbol{r} = \boldsymbol{p} \oplus (\boldsymbol{q} \oplus \boldsymbol{r})$

iii. Existence of an additive identity in *V*, i.e. there exists an additive identity (called **0**) in *V* so that for every ***p*** in *V* we have $\boldsymbol{p} \oplus \mathbf{0} = \boldsymbol{p}$

iv. Existence of additive inverses in *V*, i.e. for each ***p*** in *V* there exists an additive inverse (called **-*p***) in *V* so that $\boldsymbol{p} \oplus -\boldsymbol{p} = \mathbf{0}$

Which of the above axioms are **not** satisfied by V with its defined "addition" and scalar multiplication?

2. (9 pts) Consider the following subsets of $\mathbb{R}^4$:

i. $\{(x_1, x_2, x_3, x_4)^T \mid x_1x_3 + x_2x_4 = x_1x_2 + x_3x_4\}$

ii. $\{(x_1, x_2, x_3, x_4)^T \mid x_1 + 2x_3 = 3x_2 + 4x_4\}$

iii. $\{(x_1, x_2, x_3, x_4)^T \mid x_1 = 0 \text{ or } x_2 = 0 \text{ or } x_3 = 0 \text{ or } x_4 = 0\}$

Which of the above subsets of $\mathbb{R}^4$ are subspaces of $\mathbb{R}^4$?

3. (9 pts) Consider the following subsets of P_4:

i. $\{p \in P_4 \mid p \text{ is an even function and } p(-1) = -1\}$

ii. $\{p \in P_4 \mid p(0) = p(1)\}$

iii. $\{p \in P_4 \mid p'(1) = 0\}$

Which of the above subsets of P_4 are subspaces of P_4?

4. (12 pts) Consider the following subsets of $\mathbb{R}^3$:

i. $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ ii. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$

iii. $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ iv. $\left\{ \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \right\}$

Which of the above subsets of $\mathbb{R}^3$ are spanning sets for $\mathbb{R}^3$?

5. (12 pts) Consider the following subsets of $\mathbb{R}^3$:

i. $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$ ii. $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

iii. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ iv. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

Which of the above subsets of $\mathbb{R}^3$ are linearly independent?

6. (6 pts) Consider the following subsets P_3.

i. $S = \{1 + x, x + x^2, 1 + x^2\}$ ii. $S = \{1 + 2x - x^2, 1 + x - x^2, 1 + 3x - x^2\}$

Which of the above subsets of P_3 are linearly independent?

7. (9 pts) Consider the following subsets of $\mathbb{R}^2$:

i. $\left\{\begin{bmatrix}-6\\12\end{bmatrix},\begin{bmatrix}4\\-8\end{bmatrix}\right\}$ ii. $\left\{\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}-1\\-1\end{bmatrix}\right\}$

iii. $\left\{\begin{bmatrix}1\\3\end{bmatrix},\begin{bmatrix}-2\\-4\end{bmatrix},\begin{bmatrix}3\\-2\end{bmatrix}\right\}$

Which of the above subsets of $\mathbb{R}^2$ form a basis for $\mathbb{R}^2$?

8. (12 pts) Consider the following subsets of $\mathbb{R}^3$:

i. $\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix},\begin{bmatrix}2\\0\\1\end{bmatrix}\right\}$ ii. $\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix},\begin{bmatrix}-1\\-2\\-2\end{bmatrix},\begin{bmatrix}2\\1\\1\end{bmatrix}\right\}$

iii. $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}-1\\1\\-3\end{bmatrix},\begin{bmatrix}-3\\-2\\1\end{bmatrix}\right\}$ iv. $\left\{\begin{bmatrix}1\\-1\\1\end{bmatrix},\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\-3\\1\end{bmatrix},\begin{bmatrix}2\\1\\1\end{bmatrix}\right\}$

Which of the above subsets of $\mathbb{R}^3$ forms a basis for $\mathbb{R}^3$?

9. (3 pts) Consider the following statement:

Theorem. Let $\{\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{a}_3,\cdots,\boldsymbol{a}_n\}\subset\mathbb{R}^n$ and let $A=[\boldsymbol{a}_1\ \boldsymbol{a}_2\ \boldsymbol{a}_3\ \cdots\ \boldsymbol{a}_n]$. The following are equivalent:

i. A is invertible
ii. $A\boldsymbol{x}=\boldsymbol{0}$ has a unique solution
iii. A is row equivalent to I
iv. $A\boldsymbol{x}=\boldsymbol{b}$ has a solution for every $\boldsymbol{b}\in\mathbb{R}^n$
v. $\det(A)\neq 0$
vi. $\operatorname{diag}(A)\neq 0$
vii. $\{\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{a}_3,\cdots,\boldsymbol{a}_n\}$ is a basis for $\mathbb{R}^n$
viii. $\dim(span(\{\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{a}_3,\cdots,\boldsymbol{a}_n\}))=n$
ix. $N(A)=\{\boldsymbol{0}\}$

Which one of the above statements needs to be deleted to make the theorem true?

10. (3 pts) Let A be a 8×16 matrix. If the dimension of the null space of A is 11, then the rank of A is:

a. 3 b. 4 c. 5 d. 8
e. Undeterminable from the given information

Section II Answer the problem in this section on back of the answer sheet. You do <u>not</u> need to rewrite the problem statement. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

11. (15 pts) Consider the matrix A given by $A = \begin{bmatrix} -1 & 2 & -1 & 3 & 1 \\ 1 & 1 & -11 & 9 & -4 \\ 1 & -1 & -3 & 1 & -2 \\ -3 & 4 & 5 & 1 & 5 \end{bmatrix}$. A straightforward reduction by elimination shows that A is row equivalent to U where

$$U = \begin{bmatrix} 1 & 0 & -7 & 5 & -3 \\ 0 & 1 & -4 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. Find a basis for the row space of A.
b. Find a basis for the column space of A.
c. Find a basis for the null space of A.

Answer Sheet - Form B

Name ___________________________

1.
i. Is Not Satisfied Is Satisfied ii. Is Not Satisfied Is Satisfied

iii. Is Not Satisfied Is Satisfied iv. Is Not Satisfied Is Satisfied

2.
i. Yes No ii. Yes No iii. Yes No

3.
i. Yes No ii. Yes No iii. Yes No

4.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

5.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

6.
i. Yes No ii. Yes No

7.
i. Yes No ii. Yes No iii. Yes No

8.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

9. ______

10. ______