MATH 2360-001 Exam I 28 September 2009

Form B

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Calculators are not permitted. Do your own work. **Show all relevant supporting steps!**

1. (6 pts) Identify which of the following matrices are

(i) in reduced row echelon form [RREF],
(ii) in row echelon form but not reduced row echelon form [ROW],
(iii) not in row echelon form [NOT].

Use the labels RREF, ROW and NOT to denote your answers. (Note options (i), (ii) and (iii) are mutually exclusive.)

a. $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ e. $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ f. $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. (3 pts) Each of the following augmented matrices is in row echelon form.

For each case, indicate whether the corresponding system of linear equations is consistent or is inconsistent

a. $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$

3. (3 pts) Each of the following augmented matrices from Problem 2 and re-given below is in row echelon form.

For each case in which the corresponding system of linear equations is consistent, indicate whether the system has a unique solution or infinitely many solutions.

a. $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$

4. (3 pts) Each of the following augmented matrices from Problem 2 and re-given below is in row echelon form.

For each case in which the corresponding system of linear equations is consistent and has a unique solution, find that unique solution.

a. $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$

5. (12 pts) Each of the following augmented matrices is in reduced row echelon form. For each case, find the complete solution set of the corresponding system of linear equations.

a. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

6. (10 pts) Consider the following system of linear equations.

A. Construct an augmented matrix to represent the system of linear equations.
B. Use Gaussian elimination to transform the augmented matrix to a matrix in row echelon form. State explicitly the specific elementary row operation which is being done at each step of the Gaussian elimination.
C. Do NOT solve the system of equations.

$$\begin{cases} x_1 \phantom{{}+x_2} + x_3 + x_4 = 1 \\ -2x_1 + x_2 - 3x_3 + x_4 = 0 \\ \phantom{-2x_1+{}} x_2 \phantom{{}-3x_3} - x_4 = -1 \end{cases}$$

7. (4 pts) Consider the matrices

$$A = \begin{bmatrix} -1 & 1 & -2 \\ 1 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}$$

For each of the following operations, indicate whether it is possible or not.

a. C + 2A b. DC c. AC d. CD^T

8. (12 pts) Consider the matrices given in Problem 7 and re-given below

$$A = \begin{bmatrix} -1 & 1 & -2 \\ 1 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}$$

For each of the following operations which is possible, perform it.

a. C + 2A b. DC c. AC d. CD^T

9. (8 pts) Let $A = \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}$. Find 2×2 matrices B and C such that $B \neq C$ and neither is the zero matrix for which the matrix equation AB = AC holds.

10. (8 pts) For each of the following pairs of matrices find an elementary matrix E such that EA = B.

a. $A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -8 \\ 3 & -1 \end{bmatrix}$

b. $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & -1 \\ 2 & -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & -1 \\ -1 & -1 & 4 \end{bmatrix}$

11. (14 pts) Find the determinant of each of the following matrices

a. $A = \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix}$ b. $B = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 3 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

c. $C = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 1 & -1 & 3 \\ -2 & 1 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

12. (3 pts) For each of the matrices in Problem 11 and re-given below, determine whether it is singular or non-singular.

a. $A = \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix}$ b. $B = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 3 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

c. $C = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 1 & -1 & 3 \\ -2 & 1 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

13. (9 pts) Let A and B be 3×3 matrices such that det(A) = -2 and det(B) = 5. Find the value of

a. det(BA) b. det(2B) c. $\det(B^2)$

14. (8 pts) Find all values of c for which the following matrix is singular

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & -1 & c \\ 4 & c & -2 \end{bmatrix}$$