Math 2360 — Exam III — 30 July 2008

Form B - Make Up

Section I For each of the problems in the this section at least one of the choices is correct. For some of the problems more than one of the choices is correct. Record your answers for problems in this section on the answer sheet (last page).

1. (3 pts) Let A be a 5×8 matrix. If the dimension of the null space of A is 3, then the rank of A is:

a. 2 b. 3 c. 5 d. 6
e. Undeterminable from the given information

2. (9 pts) Determine whether the following are linear transformations from $\mathbb{R}^3$ to $\mathbb{R}^2$

a. $L(\boldsymbol{x}) = \begin{bmatrix} x_1 + x_3 - x_2 \\ x_1 + x_3 - x_2 \end{bmatrix}$ b. $L(\boldsymbol{x}) = \begin{bmatrix} 2x_1x_3 - x_2 \\ x_1 + x_3 - 2x_2 \end{bmatrix}$

c. $L(\boldsymbol{x}) = \begin{bmatrix} x_3 \\ 1 - x_1 \end{bmatrix}$

3. (9 pts) Determine whether the following are linear transformations from P_3 to P_3

a. $L(p(x)) = p(x) - p'(1)(x-1)$ b. $L(p(x)) = p(x) - p(0)$

c. $L(p(x)) = x^3 p(\frac{1}{x})$

Section II Answer the problems in this section on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

4. (15 pts) Consider the matrix A given by $A = \begin{bmatrix} 2 & -1 & 0 & 1 & -2 \\ -1 & 1 & 2 & -1 & 4 \\ 3 & -1 & 2 & 1 & 0 \end{bmatrix}$. A straightforward reduction by elimination shows that A is row equivalent to U where $U = \begin{bmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & 4 & -1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a. Find a basis for the row space of A.
b. Find a basis for the column space of A.
c. Find a basis for the null space of A.

5. (6 pts) Consider the linear transformation mapping $\mathbb{R}^4$ to $\mathbb{R}^3$ given by

$$L(\boldsymbol{x}) = (2x_2 - x_1 + 3x_4, x_1 + 3x_2 - x_3 - x_4, 2x_1 - x_2 - x_3)^T$$

Find the standard matrix representation for L.

6. (10 pts) Consider the linear transformation mapping $\mathbb{R}^4$ to $\mathbb{R}^3$ given by

$$L(\boldsymbol{x}) = (x_1 - x_2 + x_4, x_1 + 2x_2 - 3x_3, x_1 + x_3 - 2x_4)^T$$

a. Find the kernel of L.
b. Find the dimension of the range of L.

7. (10 pts) Consider the linear transformation mapping P_3 to P_3 given by
$L(p(x)) = 2p(x) - xp'(x)$

a. Find the kernel of L.
b. Find the dimension of the range of L.

8. (10 pts) Consider the linear transformation mapping P_3 to P_3 which satisfies the conditions that

$L(1+x+x^2) = 1+2x+3x^2$ $\quad$ $L(1-x-x^2) = 1+2x$ $\quad$ $L(x-x^2) = 1$

Find the value of $L(9+13x-4x^2)$

9. (12 pts) Consider the linear transformation mapping $\mathbb{R}^3$ to $\mathbb{R}^2$ given by

$$L(\boldsymbol{x}) = (x_1 + 2x_2 - 3x_3, 2x_1 - x_2)^T$$

Find a matrix A which represents L with respect the standard basis $[\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3]$ in $\mathbb{R}^3$ and the ordered basis $[\boldsymbol{b}_1, \boldsymbol{b}_2]$ in $\mathbb{R}^2$ where $b_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

10. (16 pts) Consider the linear operator on $\mathbb{R}^2$ given by

$$L(\boldsymbol{x}) = (2x_1 - x_2, x_1 - 3x_2)^T$$

Find a matrix A which represents L with respect to the standard basis $[\boldsymbol{e}_1, \boldsymbol{e}_2]$.

Find a matrix B which represents L with respect the ordered basis $[\boldsymbol{b}_1, \boldsymbol{b}_2]$ where

$b_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $b_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$.

Name ______________________ Form A

Answers

1. ______

2.

i. Yes No ii. Yes No iii. Yes No

3.

i. Yes No ii. Yes No iii. Yes No
