Math 2360 Exam II 23 July 2008

Form B

For each of the following problems at least one of the choices is correct. For many of the problems more than one of the choices is correct. Record your answers for each problem on the answer sheet (last page). Turn in (only) the answer sheet (with your answers on it) at the end of the exam. Do your own work. You may keep the exam for your own records.

1. (12 pts) Consider the following four systems of linear equations:

i. $\begin{cases} 2x_1 + 2x_2 = -1 \\ -x_1 + 3x_2 = 2 \end{cases}$

ii. $\begin{cases} x_1 - 2x_2 + 2x_3 = 0 \\ x_1 + 3x_3 = -2 \\ -x_1 + 2x_3 = 1 \end{cases}$

iii. $\begin{cases} x_1 + 2x_2 = -1 \\ -3x_1 - 6x_2 = 2 \end{cases}$

iv. $\begin{cases} x_1 - x_2 + 2x_3 = -1 \\ x_1 + 2x_2 - x_3 = 0 \end{cases}$

Which of the above fours systems of linear equations can be solved by Cramer's Rule?

2. (12 pts) Consider the set $V = \{(x_1, x_2)^T \mid x_1, x_2 \in \mathbb{R}\}$. Define "addition" by $(x_1, x_2)^T \oplus (y_1, y_2)^T = (x_1 y_1, x_2 y_2)^T$. (We will use symbol $\oplus$ for "addition" since this is not the usual addition for order pairs.) Define scalar multiplication by $\alpha(x_1, x_2) = (\alpha x_1, \alpha x_2)$. (This is usual scalar multiplication for ordered pairs.)

Recall the vector space axioms for addition for a vector space *V*.

i. Addition is commutative, i.e. for every ***x, y*** in *V* we have $\boldsymbol{x} \oplus \boldsymbol{y} = \boldsymbol{y} \oplus \boldsymbol{x}$

ii. Addition is associative, i.e., for every ***x, y, z*** in V we have $(\boldsymbol{x} \oplus \boldsymbol{y}) \oplus \boldsymbol{z} = \boldsymbol{x} \oplus (\boldsymbol{y} \oplus \boldsymbol{z})$

iii. Existence of an additive identity in *V*, i.e. there exists an additive identity (called **0**) in *V* so that for every ***x*** in *V* we have $\boldsymbol{x} \oplus \mathbf{0} = \boldsymbol{x}$

iv. Existence of additive inverses in *V*, i.e. for each ***x*** in *V* there exists an additive inverse (called **-*x***) in *V* so that $\boldsymbol{x} \oplus -\boldsymbol{x} = \mathbf{0}$

Which of the above axioms are **not** satisfied by *V* with its defined "addition" and scalar multiplication?

3. (9 pts) Consider the following subsets of $\mathbb{R}^2$:

i. $\{(x_1, x_2)^T \mid x_1 x_2 = 0\}$

ii. $\{(x_1, x_2)^T \mid x_1 - x_2 = 0\}$

iii. $\{(x_1, x_2)^T \mid x_2 = 0\}$

Which of the above subsets of $\mathbb{R}^2$ are subspaces of $\mathbb{R}^2$?

4. (9 pts) Consider the following subsets of P_3:

i. $\{p \in P_3 \mid p(-1) = p(1)\}$ ii. $\{p \in P_3 \mid \deg(p) = 2\}$

iii. $\{p \in P_3 \mid p(-1)\,p(1) = 0\}$

Which of the above subsets of P_3 are subspaces of P_3?

5. (12 pts) Consider the following subsets of $\mathbb{R}^3$:

i. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ ii. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} \right\}$

iii. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \right\}$ iv. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

Which of the above subsets of $\mathbb{R}^3$ are spanning sets for $\mathbb{R}^3$?

6. (12 pts) Consider the following subsets of $\mathbb{R}^3$:

i. $\left\{ \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$ ii. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

iii. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix} \right\}$ iv. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix} \right\}$

Which of the above subsets of $\mathbb{R}^3$ are linearly independent?

7. (5 pts) Consider the set $S = \{1, x^2, x^2 - 2\}$ which is a subset of P_3. Is the set S linear independent?

8. (5 pts) Consider the set $S = \{1, \cos \pi x, \sin \pi x\}$ which is a subset of $C^2(-\infty, \infty)$. Is the set S linear independent?

9. (9 pts) Consider the following subsets of $\mathbb{R}^2$:

i. $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \end{bmatrix} \right\}$ ii. $\left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \end{bmatrix} \right\}$

iii. $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\}$

Which of the above subsets of $\mathbb{R}^2$ form a basis for $\mathbb{R}^2$?

10. (12 pts) Consider the following subsets of $\mathbb{R}^3$:

i. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ ii. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 6 \end{bmatrix} \right\}$

iii. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} \right\}$ iv. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$

Which of the above subsets of $\mathbb{R}^3$ forms a basis for $\mathbb{R}^3$?

11. (4 pts) Consider the following statement:

Theorem. Let $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \cdots, \boldsymbol{a}_n\} \subset \mathbb{R}^n$ and let $A = [\boldsymbol{a}_1\ \boldsymbol{a}_2\ \boldsymbol{a}_3\ \cdots\ \boldsymbol{a}_n]$. The following are equivalent:

i. A is invertible
ii. $A\boldsymbol{x} = \boldsymbol{0}$ has a unique solution
iii. A is row equivalent to I
iv. $A\boldsymbol{x} = \boldsymbol{b}$ has a solution for every $\boldsymbol{b} \in \mathbb{R}^n$
v. $\det(A) \neq 0$
vi. $\operatorname{tr}(A) \neq 0$
vii. $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \cdots, \boldsymbol{a}_n\}$ is a basis for $\mathbb{R}^n$
viii. $\dim(span(\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \cdots, \boldsymbol{a}_n\})) = n$
ix. $N(A) = \{\boldsymbol{0}\}$

Which one of the above statements needs to be deleted to make the theorem true?

Name ___________________________ Form B

Answers

1.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

2.
i. Is Not Satisfied Is Satisfied ii. Is Not Satisfied Is Satisfied

iii. Is Not Satisfied Is Satisfied iv. Is Not Satisfied Is Satisfied

3.
i. Yes No ii. Yes No iii. Yes No

4.
i. Yes No ii. Yes No iii. Yes No

5.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

6.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

7.
i. Yes No

8.
i. Yes No

9.
i. Yes No ii. Yes No iii. Yes No

10.
i. Yes No ii. Yes No iii. Yes No iv. Yes No

11. ______