

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (12 pts) Each of the following augmented matrices is in row echelon form.

- A. For each case, indicate whether the corresponding system of linear equations is consistent or inconsistent
- B. For each case in which the corresponding system of linear equations is consistent, indicate whether the system has a unique solution or infinitely many solutions.
- C. For each case in which the corresponding system of linear equations is consistent and has a unique solution, find that unique solution.

a. 
$$\left[ \begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

b. 
$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

c. 
$$\left[ \begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

2. (10 pts) Each of the following augmented matrices is in reduced row echelon form. For each case, find the solution set of the corresponding system of linear equations.

a. 
$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

b. 
$$\left[ \begin{array}{ccc|c} 1 & -3 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

3. (10 pts) Consider the following system of linear equations.

- A. Construct an augmented matrix to represent the system of linear equations.
- B. Use Gaussian elimination to transform the augmented matrix to a matrix in row echelon form. State explicitly the specific elementary row operation which is being done at each step of the Gaussian elimination.
- C. Do NOT solve the system of equations.

$$\begin{cases} x_1 - x_3 - 2x_4 = -1 \\ 2x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_2 - x_4 = 1 \end{cases}$$

4. (10 pts) Consider the matrices

$$A = \begin{bmatrix} -1 & -2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 & -2 \\ 0 & -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 3 \\ -1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$$

A. For each of the following operations, indicate whether it is possible or not.

B. For each of the following operations which is possible, perform it.

a.  $2A + B$       b.  $BC$       c.  $DA$       d.  $B^T D$

5. (8 pts) Let  $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ . Find  $2 \times 2$  matrices  $B$  and  $C$  such that  $B \neq C$  and neither is the zero matrix for which the matrix equation  $AB = AC$  holds.

6. (8 pts) For each of the following pairs of matrices find an elementary matrix  $E$  such that  $EA = B$ .

a.  $A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -6 \\ 3 & -1 \end{bmatrix}$

b.  $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & -1 \\ 2 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 & 2 \\ 0 & -1 & 1 \\ 2 & -1 & 2 \end{bmatrix}$

7. (10 pts) Using an augmented matrix, find the inverse of the matrix  $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & 1 \\ 2 & -1 & -2 \end{bmatrix}$ .

8. (12 pts) Find the determinant of each of the following matrices

a.  $A = \begin{bmatrix} 1 & -2 \\ -2 & 2 \end{bmatrix}$       b.  $B = \begin{bmatrix} -1 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

c.  $C = \begin{bmatrix} 0 & -1 & 2 & -1 \\ 1 & 1 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

9. (3 pts) For each of the matrices in problem 8 (re-given below), determine whether it is singular or non-singular.

a.  $A = \begin{bmatrix} 1 & -2 \\ -2 & 2 \end{bmatrix}$       b.  $B = \begin{bmatrix} -1 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

c.  $C = \begin{bmatrix} 0 & -1 & 2 & -1 \\ 1 & 1 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

10. (9 pts) Let A and B be  $3 \times 3$  matrices such that  $\det(A) = 4$  and  $\det(B) = 3$ . Find the value of

a.  $\det(BA)$       b.  $\det(2B)$       c.  $\det(A^2)$

11. (8 pts) Find all values of  $c$  for which the following matrix is singular

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 7 & c \\ -1 & c & 1 \end{bmatrix}$$