MATH 2360-002 Exam I July 16, 2008

Form A

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (12 pts) Each of the following augmented matrices is in row echelon form.

   A. For each case, indicate whether the corresponding system of linear equations is consistent or inconsistent
   B. For each case in which the corresponding system of linear equations is consistent, indicate whether the system has a unique solution or infinitely many solutions.
   C. For each case in which the corresponding system of linear equations is consistent and has a unique solution, find that unique solution.

a. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$

2. (10 pts) Each of the following augmented matrices is in reduced row echelon form. For each case, find the solution set of the corresponding system of linear equations.

a. $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$

3. (10 pts) Consider the following system of linear equations.

   A. Construct an augmented matrix to represent the system of linear equations.
   B. Use Gaussian elimination to transform the augmented matrix to a matrix in row echelon form. State explicitly the specific elementary row operation which is being done at each step of the Gaussian elimination.
   C. Do NOT solve the system of equations.

$$\begin{cases} x_1 - x_3 - x_4 = 1 \\ 2x_1 + x_2 - 4x_3 + x_4 = 0 \\ x_2 - x_4 = -2 \end{cases}$$

4. (10 pts) Consider the matrices

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}$$

A. For each of the following operations, indicate whether it is possible or not.
B. For each of the following operations which is possible, perform it.

a. A + 2B b. AC c. DB d. $B^T C$

5. (8 pts) Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$. Find 2×2 matrices B and C such that $B \neq C$ and neither is the zero matrix for which the matrix equation AB = AC holds.

6. (8 pts) For each of the following pairs of matrices find an elementary matrix E such that EA = B.

a. $A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ 6 & -2 \end{bmatrix}$

b. $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & -1 \\ 2 & -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

7. (10 pts) Using an augmented matrix, find the inverse of the matrix $A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & -1 \\ 2 & -1 & -2 \end{bmatrix}$.

8. (12 pts) Find the determinant of each of the following matrices

a. $A = \begin{bmatrix} 1 & -2 \\ 2 & -2 \end{bmatrix}$ b. $B = \begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

c. $C = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 1 & -1 & 1 \\ -2 & -2 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

9. (3 pts) For each of the matrices in problem 8 (re-given below), determine whether it is singular or non-singular.

a. $A = \begin{bmatrix} 1 & -2 \\ 2 & -2 \end{bmatrix}$ b. $B = \begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

c. $C = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 1 & -1 & 1 \\ -2 & -2 & 1 & -1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

10. (9 pts) Let A and B be 3×3 matrices such that det(A) = 3 and det(B) = 5. Find the value of

a. det(BA) b. det(2B) c. $\det(B^2)$

11. (8 pts) Find all values of *c* for which the following matrix is singular

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 7 & c \\ 2 & c & 1 \end{bmatrix}$$