Applications of Integration to Problems

Problems = Area between curves
Volume of solids
Area in polar coordinates
Work
Fluid force
Moments
Centroids

Process

a. Subdivide the problem into smaller subproblems

\[ P = P_1 \cup P_2 \cup P_3 \cup \cdots \cup P_n \]

b. Formulate an approximation for the value of each subproblem

\[ \text{Value}(P_k) \approx V_k = F(x_k^*) \Delta x_k \]

c. Sum up the approximate values (form a Riemann Sum)

\[ V_1 + V_2 + V_3 + \cdots + V_n = \]
\[ F(x_1^*) \Delta x_1 + F(x_2^*) \Delta x_2 + F(x_3^*) \Delta x_3 + \cdots + F(x_n^*) \Delta x_n = \]
\[ \sum_{k=1}^{n} F(x_k^*) \Delta x_k = RS \]

d. Limiting Case as \[ \| \Delta_k \| \to 0 \]

\[ RS = \sum_{k=1}^{n} F(x_k^*) \Delta x_k \to \int_{a}^{b} F(x) \, dx \]