Math 1452-006 | Competency Exam
Version B | 16 January 2015

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. For each of the following functions find the derivative:

56 pts

a. $a(x) = -\frac{6}{x^2} + 4\sqrt{x} - x^4$

$a' = \frac{12}{x^3} + 2\frac{1}{\sqrt{x}} - 4x^3$

b. $b(x) = \frac{x^2+4}{x^2-1}$

$b' = \frac{(x^2-1)2x - (x^2+4)2x}{(x^2-1)^2} = \frac{-10x}{(x^2-1)^2}$

c. $c(x) = \sin(2-5x^2)$

$c' = \cos(2-5x^2)(-10x)$

d. $d(x) = \frac{\cos x}{1-\sin x}$

$d' = \frac{(1-\sin x)(-\sin x) - (\cos x)(-\cos x)}{(1-\sin x)^2} = \frac{1}{1-\sin x}$

e. $e(x) = (x-3)e^x$

$e' = (x-3)e^x + 1\cdot e^x = (x-2)e^x$

f. $f(x) = (x^3+2x^2)\ln(x)$

$f' = (x^3+2x^2)\frac{1}{x} + (3x^2+4x)\ln x$

g. $g(x) = \tan^{-1}(4x-3)$

$g' = \frac{1}{1+(4x-3)^2}\cdot 4$

h. $h(x) = x^{1/4} + 6^x$

$h' = \frac{1}{4}x^{-3/4} + 6^x(\ln 6)$

2. Find $\frac{dy}{dx}$ by implicit differentiation: $x^4 + 3y = x^2 + y^3$

16 pts

$4x^3 + 3y' = 2x + 3y^2y' \Rightarrow 4x^3 - 2x = (3y^2-3)y' \Rightarrow y' = \frac{4x^3-2x}{3y^2-3}$

3. For each of the following find the indefinite integral:

28 pts

a. $\int (4 + 6\sqrt{x} - 5x^4)\,dx$

$= 4x + \frac{6x^{3/2}}{3/2} - x^5 + c$

b. $\int \frac{x^4+3x^2-1}{x^3}\,dx = \int \left(x + \frac{3}{x} - \frac{1}{x^3}\right)dx$

$= \frac{x^2}{2} + 3\ln|x| + \frac{1}{2x^2} + c$

c. $\int \frac{4\,dx}{\sqrt{1-x^2}}$

$= 4\sin^{-1}x + c$

d. $\int 4x\sqrt{5+x^2}\,dx$

$u = 5+x^2$

$du = 2x\,dx$

$\int 4x\sqrt{5+x^2}\,dx$

$= \int 2\sqrt{u}\,du$

$= 2\frac{u^{3/2}}{3/2} + c$

$= \frac{4}{3}(5+x^2)^{3/2} + c$