Math 1452-006 | Competency Exam Version A | 16 January 2015

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. For each of the following functions find the derivative:

56 pts

a. $a(x) = -\frac{6}{x^3} + 4\sqrt{x} - x^3$

$a' = \frac{18}{x^4} + 2\frac{1}{\sqrt{x}} - 3x^2$

b. $b(x) = \frac{x^2-4}{x^2+1}$

$b' = \frac{(x^2+1)2x - (x^2-4)2x}{(x^2+1)^2} = \frac{10x}{(x^2+1)^2}$

c. $c(x) = \cos(4-2x^3)$

$c' = -\sin(4-2x^3)(-6x^2)$

d. $d(x) = \frac{\sin x}{1+\cos x}$

$d' = \frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2} = \frac{1}{1+\cos x}$

e. $e(x) = (x+4)e^x$

$e' = (x+4)e^x + 1 \cdot e^x = (x+5)e^x$

f. $f(x) = (x^3-6x)\ln(x)$

$f' = (x^3-6x)\frac{1}{x} + (3x^2-6)\ln x$

g. $g(x) = \tan^{-1}(2+3x)$

$g' = \frac{1}{1+(2+3x)^2} \cdot 3$

h. $h(x) = x^{1/3} + 4^x$

$h' = \frac{1}{3}x^{-2/3} + 4^x(\ln 4)$

2. Find $\frac{dy}{dx}$ by implicit differentiation: $x^6 + 4y = x^3 + y^4$

16 pts

$6x^5 + 4y' = 3x^2 + 4y^3y' \Rightarrow 6x^5 - 3x^2 = (4y^3-4)y' \Rightarrow y' = \frac{6x^5-3x^2}{4y^3-4}$

3. For each of the following find the indefinite integral:

28 pts

a. $\int (6 - 8\sqrt{x} + 3x^2)\,dx$

$= 6x - \frac{8x^{3/2}}{3/2} + x^3 + C$

b. $\int \frac{x^2+3x-1}{x^3}\,dx = \int \left(\frac{1}{x} + \frac{3}{x^2} - \frac{1}{x^3}\right)dx$

$= \ln|x| - \frac{3}{x} + \frac{1}{2x^2} + C$

c. $\int \frac{3\,dx}{\sqrt{1-x^2}}$

$= 3\sin^{-1}(x) + C$

d. $\frac{1}{2}\int 2x\sqrt{9+x^2}\,dx$

$u = 9+x^2$

$du = 2x\,dx$

$\int x\sqrt{9+x^2}\,dx$

$= \frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{\cancel{2}}\frac{u^{3/2}}{\frac{3}{\cancel{2}}} + C$

$= \frac{1}{3}(9+x^2)^{3/2} + C$