5.5 PROBLEM SET

A *Problems 1–8 present pairs of integration problems, one of which will require substitution and one of which will not. As you are working these problems, think about when substitution may be appropriate.*

1. **a.** $\int_0^4 (2t+4)\,dt$ **b.** $\int_0^4 (2t+4)^{-1/2}\,dt$
2. **a.** $\int_0^{\pi/2} \sin\theta\,d\theta$ **b.** $\int_0^{\pi/2} \sin 2\theta\,d\theta$
3. **a.** $\int_0^{\pi} \cos t\,dt$ **b.** $\int_0^{\sqrt{\pi}} t\cos t^2\,dt$
4. **a.** $\int_0^4 \sqrt{x}\,dx$ **b.** $\int_{-4}^0 \sqrt{-x}\,dx$
5. **a.** $\int_0^{16} \sqrt[4]{x}\,dx$ **b.** $\int_{-16}^0 \sqrt[4]{-x}\,dx$
6. **a.** $\int x(3x^2-5)\,dx$ **b.** $\int x(3x^2-5)^{50}\,dx$
7. **a.** $\int x^2\sqrt{2x^3}\,dx$ **b.** $\int x^2\sqrt{2x^3-5}\,dx$
8. **a.** $\int \dfrac{dx}{\sqrt{1-x^2}}$ **b.** $\int \dfrac{x\,dx}{\sqrt{1-x^2}}$

Use substitution to evaluate the indefinite integrals in Problems 9–34.

9. $\int (2x+3)^4\,dx$
10. $\int \sqrt{3t-5}\,dt$
11. $\int [\tan(x^2+5x+3)(2x+5)]\,dx$
12. $\int (11-2x)^{-4/5}\,dx$
13. $\int (x^2-\cos 3x)\,dx$
14. $\int \csc^2 5t\,dt$
15. $\int \sin(4-x)\,dx$
16. $\int \cot[\ln(x^2+1)]\dfrac{2x\,dx}{x^2+1}$
17. $\int \sqrt{t}(t^{3/2}+5)^3\,dt$
18. $\int \dfrac{(6x-9)\,dx}{(x^2-3x+5)^3}$
19. $\int x\sin(3+x^2)\,dx$
20. $\int \sin^3 t\cos t\,dt$
21. $\int \dfrac{x\,dx}{2x^2+3}$
22. $\int \dfrac{x^2\,dx}{x^3+1}$
23. $\int x\sqrt{2x^2+1}\,dx$
24. $\int \dfrac{4x\,dx}{2x+1}$
25. $\int \sqrt{x}e^{x\sqrt{x}}\,dx$
26. $\int \dfrac{e^{\sqrt[3]{x}}\,dx}{x^{2/3}}$
27. $\int x(x^2+4)^{1/2}\,dx$
28. $\int x^3(x^2+4)^{1/2}\,dx$
29. $\int \dfrac{\ln x}{x}\,dx$
30. $\int \dfrac{\ln(x+1)}{x+1}\,dx$
31. $\int \dfrac{dx}{\sqrt{x}(\sqrt{x}+7)}$
32. $\int \dfrac{dx}{x^{2/3}(\sqrt[3]{x}+1)}$
33. $\int \dfrac{e^t\,dt}{e^t+1}$
34. $\int \dfrac{e^{\sqrt{t}}\,dt}{\sqrt{t}(e^{\sqrt{t}}+1)}$

Evaluate the definite integrals given in Problems 35–44. Approximate the answers to Problems 43 and 44 to two significant digits.

35. $\int_0^1 \dfrac{5x^2\,dx}{2x^3+1}$
36. $\int_1^4 \dfrac{e^{-\sqrt{x}}\,dx}{\sqrt{x}}$
37. $\int_{-\ln 2}^{\ln 2} \dfrac{1}{2}(e^x-e^{-x})\,dx$
38. $\int_0^2 (e^x-e^{-x})^2\,dx$
39. $\int_1^2 \dfrac{e^{1/x}}{x^2}\,dx$
40. $\int_0^2 x\sqrt{2x+1}\,dx$
41. $\int_0^{\pi/6} \tan 2x\,dx$
42. $\int_0^1 x^2(x^3+9)^{1/2}\,dx$
43. $\int_0^5 \dfrac{0.58}{1+e^{-0.2x}}\,dx$
44. $\int_0^{12} \dfrac{5{,}000}{1+10e^{-t/5}}\,dt$

45. HISTORICAL QUEST Johann Peter Gustav Lejeune Dirichlet was a professor of mathematics at the University of Berlin and is known for his role in formulating a rigorous foundation for calculus. He was not known as a good teacher. His nephew wrote that the mathematics instruction he received from Dirichlet was the most dreadful experience of his life. Howard Eves tells of the time Dirichlet was to deliver a lecture on definite integrals, but because of illness he posted the following note:

LEJEUNE DIRICHLET
1805–1859

> *Because of illness I*
> *cannot lecture today*
> *Dirichlet*

The students then doctored the note to read as follows:

> *Michaelmas*
> $\int$
> *Easter*
> *Because of illness I*
> *cannot lecture today* *d (1 Frdor)*
> *Dirichlet*

Michaelmas and Easter were school holidays, and 1 Frdor (Friedrichsd'or) was the customary honorarium for a semester's worth of lectures. ■

a. What is the answer when you integrate the student-doctored note?

b. The so-called *Dirichlet function* is often used for counterexamples in calculus. Look up the definition of this function. What special property does it have?

B *Find the area of the region under the curves given in Problems 46–49.*

46. $y = t\sqrt{t^2+9}$ on $[0,4]$
47. $y = \dfrac{1}{t^2}\sqrt{5-\dfrac{1}{t}}$ on $\left[\frac{1}{5},1\right]$