5.5 PROBLEM SET

Problems 1–8 present pairs of integration problems, one of which will require substitution and one of which will not. As you are working these problems, think about when substitution may be appropriate.

1. a. \[ \int_0^1 (2t + 4) \, dt \]  b. \[ \int_0^1 (2t + 4)^{-1/2} \, dt \]

2. a. \[ \int_0^{\pi/2} \sin \theta \, d\theta \]  b. \[ \int_0^{\pi/2} \sin 2\theta \, d\theta \]

3. a. \[ \int_0^1 \cos t \, dt \]  b. \[ \int_0^1 t \cos t^2 \, dt \]

4. a. \[ \int_0^4 \sqrt{x} \, dx \]  b. \[ \int_{-4}^4 \sqrt{-x} \, dx \]

5. a. \[ \int_0^1 \sqrt{x} \, dx \]  b. \[ \int_{-1}^0 \sqrt{-x} \, dx \]

6. a. \[ \int_0^{2x^2 - 5} \, dx \]  b. \[ \int_0^{x^2 - 5} \, dx \]

7. a. \[ \int_0^1 \sqrt{2x^3} \, dx \]  b. \[ \int_0^1 \frac{x \, dx}{\sqrt{1 - x^2}} \]

8. a. \[ \int_0^1 \sqrt{2x^3} \, dx \]  b. \[ \int_0^1 \frac{1}{x} \, dx \]

9. \[ \int_2^3 (x + 3)^4 \, dx \]  10. \[ \int_0^{\sqrt{3} - 5} \, dt \]

11. \[ \int_0^{(x^2 - 5x + 3)(2x + 5)} \, dx \]  12. \[ \int_0^{(11 - 2x)^{-2/5}} \, dx \]

13. \[ \int_0^{\cos^2 5t} \, dt \]  14. \[ \int_0^{\csc^2 5t} \, dt \]

15. \[ \int_0^{1 - x} \, dx \]  16. \[ \int_0^{\cot \ln(x^2 + 1)} \, dx \]

17. \[ \int_0^{x^2 + 3/4} \, dx \]  18. \[ \int_0^{(x^2 - 3x + 5)^3} \, dx \]

19. \[ \int_0^{x \sin(3 + x^2)} \, dx \]  20. \[ \int_0^{x \cos t \, dt} \]

21. \[ \int_0^{x^2 + 3} \, dx \]  22. \[ \int_0^{x^2 + 3} \, dx \]

23. \[ \int_0^{x^2 + 3} \, dx \]  24. \[ \int_0^{x^2 + 3} \, dx \]

25. \[ \int_0^{x^2} \sqrt{e^{2x}} \, dx \]  26. \[ \int_0^{e^{2x}} \sqrt{e^{2x}} \, dx \]

27. \[ \int_0^{x^2 + 4} \frac{1/2} {dx} \]  28. \[ \int_0^{x^2 + 4} \frac{1/2} {dx} \]

29. \[ \int_0^{\ln x} \frac{1} {dx} \]  30. \[ \int_0^{\ln(x + 1)} \frac{1} {dx} \]

31. \[ \int_0^{\ln(x + 7)} \frac{1} {dx} \]  32. \[ \int_0^{\ln(x + 1)} \frac{1} {dx} \]

33. \[ \int_0^{e^t} \frac{1} {e^t + 1} \]  34. \[ \int_0^{e^t} \frac{1} {e^t + 1} \]

Use substitution to evaluate the indefinite integrals in Problems 9–34.

35. \[ \int_0^1 \frac{5x^2} {2x^2 + 1} \, dx \]  36. \[ \int_0^1 \frac{e^{x^2}} {\sqrt{x}} \, dx \]

37. \[ \int_0^1 \frac{1} {\ln x} \, dx \]  38. \[ \int_0^1 \frac{e^x - e^{-x}} {2} \, dx \]

39. \[ \int_0^1 \frac{e^{1/5}} {x} \, dx \]  40. \[ \int_0^1 \frac{x \ln 2x + 1} {dx} \]

41. \[ \int_0^1 \frac{\tan x} {dx} \]  42. \[ \int_0^1 \frac{x^2(3x + 5)^{1/2}} {dx} \]

43. \[ \int_0^1 \frac{0.58} {1 + e^{-0.2x}} \, dx \]  44. \[ \int_0^1 \frac{5000} {1 + 10e^{-0.5x}} \, dx \]

45. HISTORICAL QUEST

Johann Peter Gustav Lejeune Dirichlet was a professor of mathematics at the University of Berlin and is known for his role in formulating a rigorous foundation for calculus. He was not known as a good teacher. His nephew wrote that the mathematics instruction he received from Dirichlet was the most dreadful experience of his life. Howard Eves tells of the time Dirichlet was to deliver a lecture on definite integrals, but because of illness he posted the following note:

Because of illness 9 cannot lecture today.

Michaelmas

Easter

Because of illness 9 cannot lecture today

a (1 Frdor)

Find the area of the region under the curve given in Problems 46–49.

46. \[ y = r \sqrt{r^2 + 9} \, dx \] on \([0, 4]\)

47. \[ y = \frac{r}{r^2 + 1} \, dx \] on \([\frac{1}{2}, 1]\)