5.4 PROBLEM SET

A *In Problems 1–30, evaluate the definite integral.*

1. $\int_{-10}^{10} 7\,dx$
2. $\int_{-5}^{7} (-3)\,dx$
3. $\int_{-3}^{5} (2x + a)\,dx$
4. $\int_{-2}^{2} (b - x)\,dx$
5. $\int_{-1}^{2} ax^3\,dx$
6. $\int_{-1}^{1} (x^3 + bx^2)\,dx$
7. $\int_{1}^{2} \frac{c}{x^3}\,dx$
8. $\int_{-2}^{-1} \frac{p}{x^2}\,dx$
9. $\int_{0}^{9} \sqrt{x}\,dx$
10. $\int_{0}^{27} \sqrt[3]{x}\,dx$
11. $\int_{0}^{1} (5u^7 + \pi^2)\,du$
12. $\int_{0}^{1} (7x^8 + \sqrt{\pi})\,dx$
13. $\int_{1}^{2} x^{2a}\,dx,\ a \neq -\frac{1}{2}$
14. $\int_{1}^{2} (2x)^{\pi}\,dx$
15. $\int_{\ln 2}^{\ln 5} 5e^x\,dx$
16. $\int_{e^{-2}}^{e} \frac{dx}{x}$
17. $\int_{0}^{4} \sqrt{x}(x + 1)\,dx$
18. $\int_{0}^{1} \sqrt{t}(t - \sqrt{t})\,dt$
19. $\int_{1}^{2} \frac{x^3 + 1}{x^2}\,dx$
20. $\int_{1}^{4} \frac{x^2 + x - 1}{\sqrt{x}}\,dx$
21. $\int_{1}^{\sqrt{3}} \frac{6a}{1 + x^2}\,dx$
22. $\int_{0}^{0.5} \frac{b\,dx}{\sqrt{1 - x^2}}$
23. $\int_{-2}^{3} (\sin^2 x + \cos^2 x)\,dx$
24. $\int_{0}^{\pi/4} (\sec^2 x - \tan^2 x)\,dx$
25. $\int_{0}^{1} (1 - e^t)\,dt$
26. $\int_{1}^{2} \frac{x^3 + 1}{x}\,dx$
27. $\int_{0}^{1} \frac{x^2 - 4}{x - 2}\,dx$
28. $\int_{0}^{1} \frac{x^2 - 1}{x^2 + 1}\,dx$
29. $\int_{-1}^{2} (x + |x|)\,dx$
30. $\int_{0}^{2} (x - |x - 1|)\,dx$

In Problems 31–38, find the area of the region under the given curve over the prescribed interval.

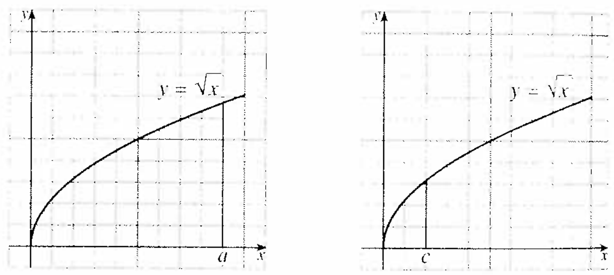
31. $y = x^2 + 1$ on $[-1, 1]$
32. $y = \sqrt{t}$ on $[0, 1]$
33. $y = \sec^2 x$ on $\left[0, \frac{\pi}{4}\right]$
34. $y = \sin x + \cos x$ on $\left[0, \frac{\pi}{2}\right]$
35. $y = e^t - t$ on $[0, 1]$
36. $y = (x^2 + x + 1)\sqrt{x}$ on $[1, 4]$
37. $y = \frac{x^2 - 2x + 3}{x}$ on $[1, 2]$
38. $y = \frac{2}{1 + t^2}$ on $[0, 1]$

In Problems 39–44, find the derivative of the given function.

39. $F(x) = \int_{0}^{x} \frac{t^2 - 1}{\sqrt{t} + 1}\,dt$
40. $F(x) = \int_{-2}^{x} (t + 1)\sqrt[3]{t}\,dt$
41. $F(t) = \int_{1}^{t} \frac{\sin x}{x}\,dx$
42. $F(t) = \int_{t}^{2} \frac{e^x}{x}\,dx$
43. $F(x) = \int_{x}^{1} \frac{dt}{\sqrt{1 + 3t^2}}$
44. $F(x) = \int_{\pi/3}^{x} \sec^2 t \tan t\,dt$

B *The formulas in Problems 45–50 are taken from a table of integrals. In each case, use differentiation to verify that the formula is correct.*

45. $\int \cos^2 au\,du = \frac{u}{2} + \frac{\sin 2au}{4a} + C$
46. $\int u\cos^2 au\,du = \frac{u^2}{4} + \frac{u\sin 2au}{4a} + \frac{\cos 2au}{8a^2} + C$
47. $\int \frac{u\,du}{(a^2 - u^2)^{3/2}} = \frac{1}{\sqrt{a^2 - u^2}} + C$
48. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$
49. $\int \frac{u\,du}{\sqrt{a^2 - u^2}} = -\sqrt{a^2 - u^2} + C$
50. $\int (\ln |u|)^2\,du = u(\ln |u|)^2 - 2u \ln |u| + 2u + C$
51. **Exploration Problem** What is the relationship between finding an area and evaluating an integral?
52. **Journal Problem** *FOCUS** ■ The area of the shaded region in Figure 5.21a is 8 times the area of the shaded region in Figure 5.21b. What is c in terms of a?



a. b.

Figure 5.21 Comparing areas

53. **Exploration Problem** If you use the first fundamental theorem for the following integral, you find

$$\int_{-1}^{1} \frac{dx}{x^2} = \left[-\frac{1}{x} \right] \Bigg|_{-1}^{1} = -1 - 1 = -2$$

But the function $y = \frac{1}{x^2}$ is never negative. What's wrong with this "evaluation"?

54. Evaluate

$$\int_{0}^{2} f(x)\,dx \quad \text{where} \quad f(x) = \begin{cases} x^3 & \text{if } 0 \le x < 1 \\ x^4 & \text{if } 1 \le x \le 2 \end{cases}$$

55. Evaluate

$$\int_{0}^{\pi} f(x)\,dx \quad \text{where} \quad f(x) = \begin{cases} \cos x & \text{if } 0 \le x < \pi/2 \\ x & \text{if } \pi/2 \le x \le \pi \end{cases}$$

*February 1995, p. 15.