3.6 PROBLEM SET

A *Find* $\frac{dy}{dx}$ *by implicit differentiation in Problems 1–14.*

1. $x^2 + y^2 = 25$
2. $x^2 + y = x^3 + y^3$
3. $xy = 25$
4. $xy(2x + 3y) = 2$
5. $x^2 + 3xy + y^2 = 15$
6. $x^3 + y^3 = x + y$
7. $\frac{1}{y} + \frac{1}{x} = 1$
8. $(2x + 3y)^2 = 10$
9. $\sin(x + y) = x - y$
10. $\tan\frac{x}{y} = y$
11. $\cos xy = 1 - x^2$
12. $e^{xy} + 1 = x^2$
13. $\ln(xy) = e^{2x}$
14. $e^{xy} + \ln y^2 = x$

In Problems 15–18, find $\frac{dy}{dx}$ *in two ways:*

a. By implicit differentiation of the equation

b. By differentiating an explicit formula for y

15. $x^2 + y^3 = 12$
16. $xy + 2y = x^2$
17. $x + \frac{1}{y} = 5$
18. $xy - x = y + 2$

Find the derivative $\frac{dy}{dx}$ *in Problems 19–32.*

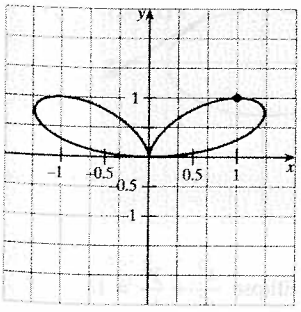
19. $y = \sin^{-1}(2x + 1)$
20. $y = \cos^{-1}(4x + 3)$
21. $y = \tan^{-1}\sqrt{x^2 + 1}$
22. $y = \cot^{-1} x^2$
23. $y = (\sin^{-1} 2x)^3$
24. $y = (\tan^{-1} x^2)^4$
25. $y = \sec^{-1}(e^{-x})$
26. $y = \ln\left|\sin^{-1} x\right|$
27. $y = \tan^{-1}\left(\frac{1}{x}\right)$
28. $y = \cos^{-1}(\sin x),\ \sin x \geq 0$
29. $y = \sin^{-1}(\cos x),\ \cos x \geq 0$
30. $y = \ln[\sin^{-1}(e^x)]$
31. $x\sin^{-1} y + y\tan^{-1} x = x$
32. $\sin^{-1} y + y = 2xy$

In Problems 33–38, find an equation of the tangent line to the graph of each equation at the prescribed point.

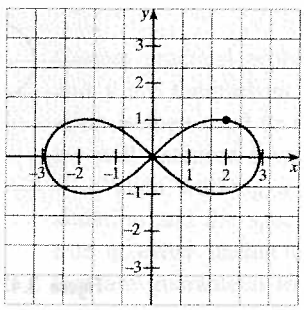
33. $x^2 + y^2 = 13$ at $(-2, 3)$
34. $x^3 + y^3 = y + 21$ at $(3, -2)$
35. $\sin(x - y) = xy$ at $(0, \pi)$
36. $3^x + \log_2(xy) = 10$ at $(2, 1)$
37. $x\tan^{-1} y = x^2 + y$ at $(0, 0)$
38. $\sin^{-1}(xy) + \frac{\pi}{2} = \cos^{-1} y$ at $(1, 0)$

Find the slope of the tangent line to the graph at the points indicated in Problems 39–42.

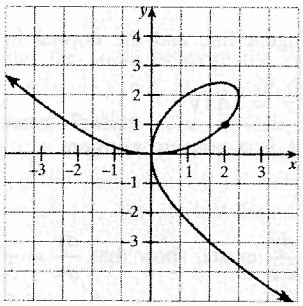
39. bifolium: $(x^2 + y^2)^2 = 4x^2y$ at $(1, 1)$



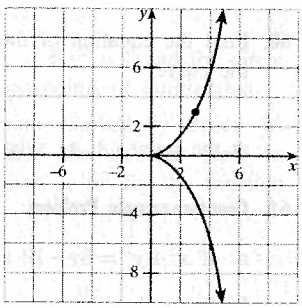
40. lemniscate of Bernoulli: $(x^2 + y^2)^2 = \frac{25}{3}(x^2 - y^2)$ at $(2, 1)$



41. folium of Descartes: $x^3 + y^3 - \frac{9}{2}xy = 0$ at $(2, 1)$



42. cissoid of Diocles: $y^2(6 - x) = x^3$ at $(3, 3)$



43. Find an equation of the normal line to the curve $x^2 + 2xy = y^3$ at $(1, -1)$.

44. Find an equation of the normal line to the curve $x^2\sqrt{y - 2} = y^2 - 3x - 5$ at $(1, 3)$.

Use implicit differentiation to find the second derivative y″ of the functions given in Problems 45–46.

45. $7x + 5y^2 = 1$
46. $x^2 + 2y^3 = 4$

B 47. **Interpretation Problem** Compare and contrast the derivatives of the following functions:

a. $y = x^2$ **b.** $y = 2^x$ **c.** $y = e^x$ **d.** $y = x^e$

48. **Interpretation Problem** Compare and contrast the derivatives of the following functions:

a. $y = \log x$ **b.** $y = \ln x$

49. **Interpretation Problem** Discuss logarithmic differentiation.

Use logarithmic differentiation in Problems 50–55 to find dy/dx. You may express your answer in terms of both x and y, and you do not need to simplify the resulting rational expressions.

50. $y = \sqrt[18]{(x^{10} + 1)^3(x^7 - 3)^8}$
51. $y = \dfrac{(2x - 1)^5}{\sqrt{x - 9}(x + 3)^2}$
52. $y = \dfrac{e^{2x}}{(x^2 - 3)^2 \ln\sqrt{x}}$
53. $y = \dfrac{e^{3x^2}}{(x^3 + 1)^2(4x - 7)^{-2}}$
54. $y = x^x$
55. $y = x^{\ln\sqrt{x}}$
56. Let $\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1$, where a and b are nonzero constants. Find

a. $\frac{du}{dv}$ **b.** $\frac{dv}{du}$

57. Show that the tangent line at the point (a, b) on the curve whose equation is $2x^2 + 3xy + y^2 = -2$ is horizontal if $4a + 3b = 0$. Find two such points on the curve.

58. Find two points on the curve whose equation is $x^2 - 3xy + 2y^2 = -2$, where the tangent line is vertical.

59. Let g be a differentiable function of x that satisfies $g(x) < 0$ and $x^2 + g^2(x) = 10$ for all x.

a. Use implicit differentiation to show that $\frac{dg}{dx} = \frac{-x}{g(x)}$.