3.5 PROBLEM SET

A 1. **WHAT DOES THIS SAY?** What is the chain rule?

2. **WHAT DOES THIS SAY?** When do you need to use the chain rule?

In Problems 3–8, use the chain rule to compute the derivative dy/dx and write your answer in terms of x only.

3. $y = u^2 + 1$; $u = 3x - 2$
4. $y = 2u^2 - u + 5$; $u = 1 - x^2$
5. $y = \frac{2}{u^2}$; $u = x^2 - 9$
6. $y = \cos u$; $u = x^2 + 7$
7. $y = u \tan u$; $u = 3x + \frac{6}{x}$
8. $y = u^2$; $u = \ln x$

Differentiate each function in Problems 9–12 with respect to the given variable of the function.

9. a. $g(u) = u^3$ b. $u(x) = x^2 + 1$
 c. $f(x) = (x^2 + 1)^3$
10. a. $g(u) = u^5$ b. $u(x) = 3x - 1$
 c. $f(x) = (3x - 1)^5$
11. a. $g(u) = u^7$ b. $u(x) = 5 - 8x - 12x^2$
 c. $f(x) = (5 - 8x - 12x^2)^7$
12. a. $g(u) = u^{15}$ b. $u(x) = 3x^2 + 5x - 7$
 c. $f(x) = (3x^2 + 5x - 7)^{15}$

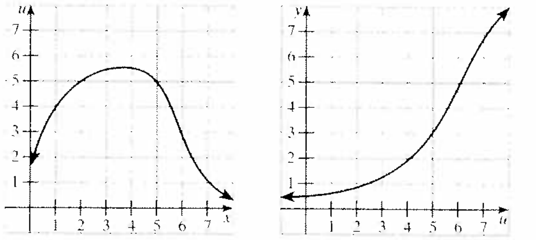
In Problems 13–40, find the derivative of the given function.

13. $f(x) = (5x - 2)^5$
14. $f(x) = (x^4 - 7x)^{15}$
15. $f(x) = (3x^2 - 2x + 1)^4$
16. $f(x) = (3 - x^2 - x^4)^{11}$
17. $s(\theta) = \sin(4\theta + 2)$
18. $c(\theta) = \cos(5 - 3\theta)$
19. $f(x) = e^{-x^2+3x}$
20. $y = e^{x^3 - \pi}$
21. $y = e^{\sec x}$
22. $g(x) = e^{\sin x}$
23. $f(t) = \exp(t^2 + t + 5)$
24. $g(t) = t^2 e^{-t} + (\ln t)^2$
25. $g(x) = x \sin 5x$
26. $h(x) = \frac{\tan 3x}{x^2}$
27. $f(x) = \left(\frac{1}{1 - 2x}\right)^3$
28. $f(x) = \sqrt[3]{\frac{1}{2 - 3x}}$
29. $f(x) = xe^{1-2x}$
30. $g(x) = \ln(3x^4 + 5x)$
31. $p(x) = \sin x^2 \cos x^2$
32. $f(x) = \csc^2(\sqrt{x})$
33. $f(x) = x^3(2 - 3x)^2$
34. $f(x) = x^4(2 - x - x^2)^3$
35. $f(x) = \sqrt{\frac{x^2 + 3}{x^2 - 5}}$
36. $f(x) = \sqrt{\frac{2x^2 - 1}{3x^2 + 2}}$
37. $f(x) = \sqrt[3]{x + \sqrt{2x}}$
38. $g(x) = \ln(\ln x)$
39. $f(x) = \ln(\sin x + \cos x)$
40. $T(x) = \ln(\sec x + \tan x)$

Find the x-coordinate of each point in Problems 41–46 where the graph of the given function has a horizontal tangent line.

41. $f(x) = x\sqrt{1 - 3x}$
42. $g(x) = x^2(2x + 3)^2$
43. $q(x) = \frac{(x - 1)^2}{(x + 2)^3}$
44. $f(x) = (2x^2 - 7)^3$
45. $T(x) = x^2 e^{1-3x}$
46. $V(x) = \frac{\ln \sqrt{x}}{x^2}$

B 47. The graphs of $u = g(x)$ and $y = f(u)$ are shown in Figure 3.29.

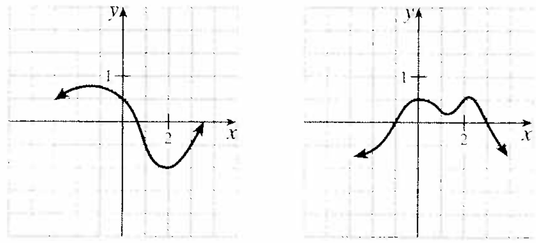


a. $u = g(x)$ b. $y = f(u)$

Figure 3.29 Find the slope of $y = f[g(x)]$.

a. Find the approximate value of u at $x = 2$. What is the slope of the tangent line at that point?
b. Find the approximate value of y at $x = 5$. What is the slope of the tangent line at that point?
c. Find the slope of $y = f[g(x)]$ at $x = 2$.

48. The graphs of $y = f(x)$ and $y = g(x)$ are shown in Figure 3.30.



a. $y = f(x)$ b. $y = g(x)$

Figure 3.30 Problem 48

Let $h(x) = f[g(x)]$.

a. Estimate $h'(-1)$ b. $h'(1)$ c. $h'(3)$

49. Repeat Problem 48 for $h(x) = g[f(x)]$.

50. If $g(x) = f[f(x)]$, use the table to find the value of $g'(2)$.

x	0.0	1.0	2.0	3.0	4.0	5.0
$f(x)$	18.5	9.4	4.0	2.6	8.3	14.0

51. If $h(x) = f[g(x)]$, use the table to find the value of $h'(1)$.

x	0.0	1.0	2.0	3.0	4.0	5.0
$f(x)$	6.9	4.3	3.1	2.8	2.2	2.0
$g(x)$	0.8	2.0	2.5	1.8	0.9	0.4

52. Assume that a spherical snowball melts in such a way that its radius decreases at a constant rate (that is, the radius is a linear function of time). Suppose it begins as a sphere with radius 10 cm and takes 2 hours to disappear.

a. What is the rate of change of its volume after 1 hour? (Recall $V = \frac{4}{3}\pi r^3$.)
b. At what rate is its surface area changing after 1 hour? (Recall $S = 4\pi r^2$.)