3.3 PROBLEM SET

A *Differentiate the functions given in Problems 1–32.*

1. $f(x) = \sin x + \cos x$
2. $f(x) = 2\sin x + \tan x$
3. $g(t) = t^2 + \cos t + \cos \frac{\pi}{4}$
4. $g(t) = 2\sec t + 3\tan t - \tan \frac{\pi}{3}$
5. $f(t) = \sin^2 t$ *Hint*: Use the product rule.
6. $g(x) = \cos^2 x$ *Hint*: Use the product rule.
7. $f(x) = \sqrt{x}\cos x + x\cot x$
8. $f(x) = 2x^3 \sin x - 3x\cos x$
9. $p(x) = x^2 \cos x$
10. $p(t) = (t^2 + 2)\sin t$
11. $q(x) = \dfrac{\sin x}{x}$
12. $r(x) = \dfrac{e^x}{\sin x}$
13. $h(t) = e^t \csc t$
14. $f(\theta) = \dfrac{\sec\theta}{2 - \cos\theta}$
15. $f(x) = x^2 \ln x$
16. $g(x) = \dfrac{\ln x}{x^2}$
17. $h(x) = e^x(\sin x - \cos x)$
18. $f(x) = \dfrac{\ln x}{x}$
19. $f(x) = \dfrac{\sin x}{e^x}$
20. $g(x) = \dfrac{x\cos x}{e^x}$
21. $f(x) = \dfrac{\tan x}{1 - 2x}$
22. $g(t) = \dfrac{1 + \sin t}{\sqrt{t}}$
23. $f(t) = \dfrac{2 + \sin t}{t + 2}$
24. $f(\theta) = \dfrac{\theta - 1}{2 + \cos\theta}$
25. $f(x) = \dfrac{\sin x}{1 - \cos x}$
26. $f(x) = \dfrac{x}{1 - \sin x}$
27. $f(x) = \dfrac{1 + \sin x}{2 - \cos x}$
28. $g(x) = \dfrac{\cos x}{1 + \cos x}$
29. $f(x) = \dfrac{\sin x + \cos x}{\sin x - \cos x}$
30. $f(x) = \dfrac{x^2 + \tan x}{3x + 2\tan x}$
31. $g(x) = \sec^2 x - \tan^2 x + \cos x$
32. $g(x) = \cos^2 x + \sin^2 x + \sin x$

Find the second derivative of each function given in Problems 33–44.

33. $f(\theta) = \sin\theta$
34. $f(\theta) = \cos\theta$
35. $f(\theta) = \tan\theta$
36. $f(\theta) = \cot\theta$
37. $f(\theta) = \sec\theta$
38. $f(\theta) = \csc\theta$
39. $f(x) = \sin x + \cos x$
40. $f(x) = x\sin x$
41. $f(x) = e^x \cos x$
42. $g(t) = t^3 e^t$
43. $h(t) = \sqrt{t}\ln t$
44. $f(t) = \dfrac{\ln t}{t}$

B *Find an equation for the tangent line at the prescribed point for each function in Problems 45–52.*

45. $f(\theta) = \tan\theta$ at $(\frac{\pi}{4}, 1)$
46. $f(\theta) = \sec\theta$ at $(\frac{\pi}{3}, 2)$
47. $f(x) = \sin x$, where $x = \frac{\pi}{6}$
48. $f(x) = \cos x$, where $x = \frac{\pi}{3}$
49. $y = x + \sin x$, where $x = 0$
50. $y = x\sec x$, where $x = 0$
51. $y = e^x \cos x$, where $x = 0$
52. $y = x\ln x$, where $x = 1$
53. Which of the following functions satisfy $y'' + y = 0$?
    a. $y_1 = 2\sin x + 3\cos x$
    b. $y_2 = 4\sin x - \pi\cos x$
    c. $y_3 = x\sin x$
    d. $y_4 = e^x \cos x$
54. For what values of A and B does $y = A\cos x + B\sin x$ satisfy $y'' + 2y' + 3y = 2\sin x$?
55. For what values of A and B does $y = Ax\cos x + Bx\sin x$ satisfy $y'' + y = -3\cos x$?

C 56. Complete the proof of Theorem 3.6 by showing that $\dfrac{d}{dx}\cos x = -\sin x$. *Hint*: You will need to use the identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

Prove the requested parts of Theorem 3.7 in Problems 57–59.

57. $\dfrac{d}{dx}\cot x = -\csc^2 x$
58. $\dfrac{d}{dx}\sec x = \sec x \tan x$
59. $\dfrac{d}{dx}\csc x = -\csc x \cot x$
60. Use the limit of a difference quotient to prove that

$$\frac{d}{dx}\tan x = \sec^2 x$$

61. **Exploration Problem** Write a short paper on the difficulties of differentiating trigonometric functions measured in degrees.*

*See, for example, "Fallacies, Flaws, and Flimflam," *The College Mathematics Journal*, Vol. 23, No. 3, May 1992, and Vol. 24, No. 4, September 1993. Another very understandable article, "Why Use Radians in Calculus?", by Carl E. Crockett, can be found in *The AMATYC Review*, Vol.19, No. 2, Spring 1998, pp. 44–47.

3.4 Rates of Change: Modeling Rectilinear Motion

IN THIS SECTION average and instantaneous rate of change, introduction to mathematical modeling, rectilinear motion (modeling in physics), falling body problems

The speed of a car or airplane; interest rates; the growth rate of a population; the drip rate of an intravenous injection—these are examples of the many situations in which rates of change are an important consideration in practical problems. In this section, we begin by showing how rates of change can be computed using differentiation. Then we