27. $f(x) = x^2 - 3x - 5$, where $x = -2$
28. $f(x) = x^5 - 3x^3 - 5x + 2$, where $x = 1$
29. $f(x) = (x^2 + 1)(1 - x^3)$, where $x = 1$
30. $f(x) = \frac{x+1}{x-1}$, where $x = 0$
31. $f(x) = \frac{x^2+5}{x+5}$, where $x = 1$
32. $f(x) = 1 - \frac{1}{x} + \frac{2}{\sqrt{x}}$, where $x = 4$

Find the coordinates of each point on the graph of the given function where the tangent line is horizontal in Problems 33–39.

33. $f(x) = 2x^3 - 7x^2 + 8x - 3$
34. $f(t) = t^4 + 4t^3 - 8t^2 + 3$
35. $g(x) = (3x - 5)(x - 8)$
36. $f(t) = \frac{1}{t^2} - \frac{1}{t^3}$
37. $f(x) = \sqrt{x}(x - 3)$
38. $h(u) = \frac{1}{\sqrt{u}}(u + 9)$
39. $h(x) = \frac{4x^2 + 12x + 9}{2x + 3}$

B 40. **a.** Differentiate the function $f(x) = 2x^2 - 5x - 3$.
b. Factor the function in part **a** and differentiate by using the product rule. Show that the two answers are the same.

41. **a.** Use the quotient rule to differentiate $f(x) = \frac{2x-3}{x^3}$.
b. Rewrite the function in part **a** as $f(x) = x^{-3}(2x - 3)$ and differentiate by using the product rule.
c. Rewrite the function in part **a** as $f(x) = 2x^{-2} - 3x^{-3}$ and differentiate.
d. Show that the answers to parts **a**, **b**, and **c** are all the same.

42. Find numbers a, b, and c that guarantee that the graph of the function $f(x) = ax^2 + bx + c$ will have x-intercepts at $(0, 0)$ and $(5, 0)$ and a tangent line with slope 1 where $x = 2$.

43. Find the equation for the tangent line to the curve with equation $y = x^4 - 2x + 1$ that is parallel to the line $2x - y - 3 = 0$.

44. Find equations for two tangent lines to the graph of $f(x) = \frac{3x+5}{1+x}$ that are perpendicular to the line $2x - y = 1$.

45. Let $f(x) = (x^3 - 2x^2)(x + 2)$.
a. Find an equation for the tangent line to the graph of f at the point where $x = 1$.
b. Find an equation for the normal line to the graph of f at the point where $x = 0$.

46. Find an equation for a normal line to the graph of $f(x) = (x^3 - 2x^2)(x + 2)$ that is parallel to the line $x - 16y + 17 = 0$.

47. Find all points (x, y) on the graph of $y = 4x^2$ with the property that the tangent line at (x, y) passes through the point $(2, 0)$.

48. Find the equations of all the tangent lines to the graph of the function $f(x) = x^2 - 4x + 25$ that pass through the origin.

Determine which (if any) of the functions $y = f(x)$ given in Problems 49–52 satisfy the equation

$$y''' + y'' + y' = x + 1$$

49. $f(x) = x^2 + 2x - 3$
50. $f(x) = x^3 + x^2 + x$
51. $f(x) = \frac{1}{2}x^2 + 3$
52. $f(x) = 2x^2 + x$

53. HISTORICAL QUEST When working with rational expressions, we need to be careful about division by zero. One of the earliest recorded treatments of division by zero is attributed to the Hindu mathematician Āryabhata (476–550), in whose honor the first Indian satellite was named. He also gave rules for approximations of square roots and sums of arithmetic progressions as well as rules for basic algebraic manipulations. One example of his work is the following calculation for π: "Add four to one hundred, multiply by eight and add again sixty-two thousand; the result is the approximate value of the circumference of a circle whose diameter is twenty-thousand."

Follow the steps of Āryabhata's approximation for π. After you have completed this demonstration, discuss the procedure and technology you used and contrast it with the tools that Āryabhata must have had available.

C 54. What is the relationship between the degree of a polynomial function P and the value of k for which $P^{(k)}(x)$ is first equal to 0?

55. Prove the constant multiple rule $(cf)' = cf'$.

56. Prove the sum rule $(f + g)' = f' + g'$.

57. Use the definition of the derivative to find the derivative of f^2, given that f is a differentiable function.

58. Prove the product rule by using the result of Problem 57 and the identity
$$fg = \tfrac{1}{2}\left[(f+g)^2 - f^2 - g^2\right]$$

59. Prove the quotient rule
$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$
where $g(x) \neq 0$. *Hint*: First show that the difference quotient for f/g can be expressed as
$$\frac{\frac{f}{g}(x + \Delta x) - \frac{f}{g}(x)}{\Delta x} = \frac{f(x+\Delta x)g(x) - f(x)g(x+\Delta x)}{(\Delta x)g(x+\Delta x)g(x)}$$
and then subtract and add the term $g(x)f(x)$ in the numerator.

60. Show that the reciprocal function $r(x) = 1/f(x)$ has the derivative $r'(x) = -f'(x)/[f(x)]^2$ at each point x where f is differentiable and $f(x) \neq 0$.

61. If f, g, and h are differentiable functions, show that the product fgh is also differentiable and
$$(fgh)' = fgh' + fg'h + f'gh$$

62. Let f be a function that is differentiable at x.
a. If $g(x) = [f(x)]^3$, show that $g'(x) = 3[f(x)]^2 f'(x)$. *Hint*: Write $g(x) = [f(x)]^2 f(x)$ and use the product rule.
b. Show that $p(x) = [f(x)]^4$ has the derivative
$$p'(x) = 4[f(x)]^3 f'(x).$$

63. Find constants A, B, and C so that $y = Ax^3 + Bx + C$ satisfies the equation
$$y''' + 2y'' - 3y' + y = x$$