			Leibniz notation		
Fourth derivative:	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	or	$\frac{d^4}{dx^4}f(x)$
⋮	⋮	⋮	⋮		⋮
***n*th derivative**	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	or	$\frac{d^n}{dx^n}f(x)$

What This Says Because the derivative of a function is a function, differentiation can be applied over and over, as long as the derivative itself is a differentiable function. That is, we can take derivatives of derivatives.

Notice also that for derivatives of higher order than the third, the parentheses distinguish a derivative from a power. For example, $f^4 \neq f^{(4)}$.

You should note that all higher-order derivatives of a polynomial $p(x)$ will also be polynomials, and if p has degree n, then $p^{(k)}(x) = 0$ for $k \geq n + 1$, as illustrated in the following example.

EXAMPLE 8 Higher-order derivatives for a polynomial function

Find the derivatives of all orders of

$$p(x) = -2x^4 + 9x^3 - 5x^2 + 7$$

Solution

$$p'(x) = -8x^3 + 27x^2 - 10x; \quad p''(x) = -24x^2 + 54x - 10;$$
$$p'''(x) = -48x + 54; \quad p^{(4)}(x) = -48; \quad p^{(5)}(x) = 0; \ldots \quad p^{(k)}(x) = 0 \ (k \geq 5)$$

3.2 PROBLEM SET

A *To demonstrate the power of the theorems of this section, Problems 1–4 ask you to go back and rework some problems in Section 3.1, using the material of this section instead of the definition of derivative.*

1. Find the derivatives of the functions given in Problems 11–16 of Problem Set 3.1.
2. Find the derivatives of the functions given in Problems 17–22 of Problem Set 3.1.
3. Find the derivatives of the functions given in Problems 23–27 of Problem Set 3.1.
4. Find the derivatives of the functions given in Problems 39–42 of Problem Set 3.1.

Differentiate the functions given in Problems 5–20. Assume that C is a constant.

5. **a.** $f(x) = 3x^4 - 9$ **b.** $g(x) = 3(9)^4 - x$
6. **a.** $f(x) = 5x^2 + x$ **b.** $g(x) = \pi^3$
7. **a.** $f(x) = x^3 + C$ **b.** $g(x) = C^2 + x$
8. **a.** $f(t) = 10t^{-1}$ **b.** $g(t) = \frac{7}{t}$
9. $r(t) = t^2 - \frac{1}{t^2} + \frac{5}{t^4}$
10. $f(x) = \pi^3 - 3\pi^2$
11. $f(x) = \frac{7}{x^2} + x^{2/3} + C$
12. $g(x) = \frac{1}{2\sqrt{x}} + \frac{x^2}{4} + C$
13. $f(x) = \frac{x^3 + x^2 + x - 7}{x^2}$
14. $g(x) = \frac{2x^5 - 3x^2 + 11}{x^3}$
15. $f(x) = (2x + 1)(1 - 4x^3)$
16. $g(x) = (x + 2)(2\sqrt{x} + x^2)$
17. $f(x) = \frac{3x + 5}{x + 9}$
18. $f(x) = \frac{x^2 + 3}{x^2 + 5}$
19. $g(x) = x^2(x + 2)^2$
20. $f(x) = x^2(2x + 1)^2$

In Problems 21–24, find f', f'', f''', and $f^{(4)}$.

21. $f(x) = x^5 - 5x^3 + x + 12$
22. $f(x) = \frac{1}{4}x^8 - \frac{1}{2}x^6 - x^2 + 2$
23. $f(x) = \frac{-2}{x^2}$
24. $f(x) = \frac{4}{\sqrt{x}}$
25. Find $\frac{d^2y}{dx^2}$, where $y = 3x^3 - 7x^2 + 2x - 3$.
26. Find $\frac{d^2y}{dx^2}$, where $y = (x^2 + 4)(1 - 3x^3)$.

In Problems 27–32, find the standard form equation for the tangent line to $y = f(x)$ at the specified point.