In each of Problems 11–16, a function \( f \) is given along with a number \( c \) in its domain.

a. Find the difference quotient of \( f \).

b. Find \( f'(c) \) by computing the limit of the difference quotient.

11. \( f(x) = 3 \) at \( c = -5 \)
12. \( f(x) = x \) at \( c = 2 \)
13. \( f(x) = 2x \) at \( c = 1 \)
14. \( f(x) = 2x^2 \) at \( c = 1 \)
15. \( f(x) = 2 - x^2 \) at \( c = 0 \)
16. \( f(x) = -x^2 \) at \( c = 2 \)

Use the definition to differentiate the functions given in Problems 17–28, and then describe the set of all numbers for which the function is differentiable.

17. \( f(x) = 5 \)
18. \( g(x) = 3x \)
19. \( f(x) = 3x - 7 \)
20. \( g(x) = 4 - 5x \)
21. \( g(x) = 3x^2 \)
22. \( h(x) = 2x^2 + 3 \)
23. \( f(x) = x^2 - x \)
24. \( g(t) = 4 - t^2 \)
25. \( f(s) = (s - 1)^2 \)
26. \( f(x) = \frac{1}{2x} \)
27. \( f(x) = \sqrt{5x} \)
28. \( f(x) = \sqrt{x + 1} \)

Find an equation for the tangent line to the graph of the function at the specified point in Problems 29–34.

29. \( f(x) = 3x - 7 \) at \( (3, 2) \)
30. \( g(x) = 3x^2 \) at \( (-2, 12) \)
31. \( f(s) = s^3 \) at \( s = -\frac{1}{2} \)
32. \( g(t) = 4 - t^2 \) at \( t = 0 \)
33. \( f(x) = \frac{1}{x + 3} \) at \( x = 2 \)
34. \( g(x) = \sqrt{x - 5} \) at \( x = 9 \)

Find an equation of the normal line to the graph of the function at the specified point in Problems 35–38.

35. \( f(x) = 3x - 7 \) at \( (3, 2) \)
36. \( g(x) = 4 - 5x \) at \( (0, 4) \)
37. \( f(x) = \frac{1}{x + 3} \) at \( x = 3 \)
38. \( f(x) = \sqrt{5x} \) at \( x = 5 \)

Find \( \frac{dy}{dx} \) for the functions and values of \( c \) given in Problems 39–42.

39. \( y = 2x \), \( c = -1 \)
40. \( y = 4 - x \), \( c = 2 \)
41. \( y = 1 - x^2 \), \( c = 0 \)
42. \( y = \frac{4}{x} \), \( c = 1 \)

43. Suppose \( f(x) = x^2 \).

a. Compute the slope of the secant line joining the points on the graph of \( f \) whose \( x \)-coordinates are \(-2\) and \(-1\).

b. Use calculus to compute the slope of the line that is tangent to the graph when \( x = -2 \) and compare this slope with your answer in part a.

44. Suppose \( f(x) = x^3 \).

a. Compute the slope of the secant line joining the points on the graph of \( f \) whose \( x \)-coordinates are \( 1 \) and \( 1.1 \).

b. Use calculus to compute the slope of the line that is tangent to the graph when \( x = 1 \) and compare this slope to your answer from part a.

45. Sketch the graph of the function \( y = x^2 - x \). Determine the value(s) of \( x \) for which the derivative is 0. What happens to the graph at the corresponding point(s)?

46. a. Find the derivative of \( f(x) = x^2 - 3x \).

b. Show that the parabola whose equation is \( y = x^2 - 3x \) has one horizontal tangent line. Find the equation of this line.

c. Find a point on the graph of \( f \) where the tangent line is parallel to the line \( 3x + y = 11 \).

d. Sketch the graph of the parabola whose equation is \( y = x^2 - 3x \). Display the horizontal tangent line and the tangent line found in part c.

47. a. Find the derivative of \( f(x) = 4 - 2x^2 \).

b. The graph of \( f \) has one horizontal tangent line. What is its equation?

c. At what point on the graph of \( f \) is the tangent line parallel to the line \( 8x + 3y = 4 \)?

48. Show that the function \( f(x) = |x - 2| \) is not differentiable at \( x = 2 \).

49. Is the function \( f(x) = 2| x + 1| \) differentiable at \( x = 1 \)?

50. Let \( f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases} \)

Does \( f'(0) \) exist? Hint: Find the difference quotient and take the limit as \( \Delta x \to 0 \) from the left and from the right.

51. Let \( f(x) = \begin{cases} -2x & \text{if } x < 1 \\ \sqrt{x - 3} & \text{if } x \geq 1 \end{cases} \)

a. Sketch the graph of \( f \).

b. Show that \( f \) is continuous but not differentiable at \( x = 1 \).

52. Counterexample Problem Give an example of a function that is continuous on \((-\infty, \infty)\) but is not differentiable at \( x = 5 \).

Estimate the derivative \( f'(c) \) in Problems 53–58 by evaluating the difference quotient

\[
\frac{\Delta y}{\Delta x} = \frac{f(c + \Delta x) - f(c)}{\Delta x}
\]

at a succession of numbers near \( c \).