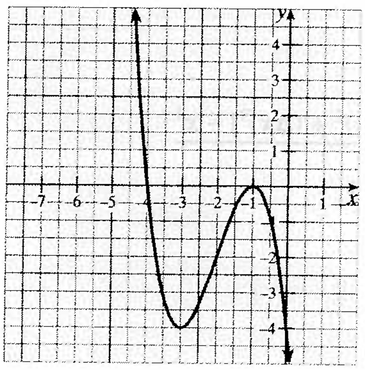
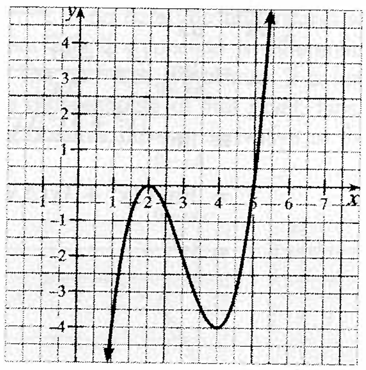
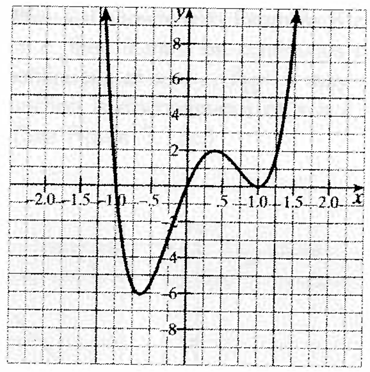
8. 

9. 

10. 

In each of Problems 11–16, a function f is given along with a number c in its domain.

a. Find the difference quotient of f.

b. Find $f'(c)$ by computing the limit of the difference quotient.

11. $f(x) = 3$ at $c = -5$ **12.** $f(x) = x$ at $c = 2$

13. $f(x) = 2x$ at $c = 1$ **14.** $f(x) = 2x^2$ at $c = 1$

15. $f(x) = 2 - x^2$ at $c = 0$ **16.** $f(x) = -x^2$ at $c = 2$

Use the definition to differentiate the functions given in Problems 17–28, and then describe the set of all numbers for which the function is differentiable.

17. $f(x) = 5$ **18.** $g(x) = 3x$

19. $f(x) = 3x - 7$ **20.** $g(x) = 4 - 5x$

21. $g(x) = 3x^2$ **22.** $h(x) = 2x^2 + 3$

23. $f(x) = x^2 - x$ **24.** $g(t) = 4 - t^2$

25. $f(s) = (s - 1)^2$ **26.** $f(x) = \dfrac{1}{2x}$

27. $f(x) = \sqrt{5x}$ **28.** $f(x) = \sqrt{x + 1}$

Find an equation for the tangent line to the graph of the function at the specified point in Problems 29–34.

29. $f(x) = 3x - 7$ at $(3, 2)$ **30.** $g(x) = 3x^2$ at $(-2, 12)$

31. $f(s) = s^3$ at $s = -\frac{1}{2}$ **32.** $g(t) = 4 - t^2$ at $t = 0$

33. $f(x) = \dfrac{1}{x + 3}$ at $x = 2$ **34.** $g(x) = \sqrt{x - 5}$ at $x = 9$

Find an equation of the normal line to the graph of the function at the specified point in Problems 35–38.

35. $f(x) = 3x - 7$ at $(3, 2)$ **36.** $g(x) = 4 - 5x$ at $(0, 4)$

37. $f(x) = \dfrac{1}{x + 3}$ at $x = 3$ **38.** $f(x) = \sqrt{5x}$ at $x = 5$

Find $\left.\dfrac{dy}{dx}\right|_{x=c}$ *for the functions and values of c given in Problems 39–42.*

39. $y = 2x$, $c = -1$ **40.** $y = 4 - x$, $c = 2$

41. $y = 1 - x^2$, $c = 0$ **42.** $y = \dfrac{4}{x}$, $c = 1$

B **43.** Suppose $f(x) = x^2$.

a. Compute the slope of the secant line joining the points on the graph of f whose x-coordinates are -2 and -1.9.

b. Use calculus to compute the slope of the line that is tangent to the graph when $x = -2$ and compare this slope with your answer in part **a.**

44. Suppose $f(x) = x^3$.

a. Compute the slope of the secant line joining the points on the graph of f whose x-coordinates are 1 and 1.1.

b. Use calculus to compute the slope of the line that is tangent to the graph when $x = 1$ and compare this slope to your answer from part **a.**

45. Sketch the graph of the function $y = x^2 - x$. Determine the value(s) of x for which the derivative is 0. What happens to the graph at the corresponding point(s)?

46. a. Find the derivative of $f(x) = x^2 - 3x$.

b. Show that the parabola whose equation is $y = x^2 - 3x$ has one horizontal tangent line. Find the equation of this line.

c. Find a point on the graph of f where the tangent line is parallel to the line $3x + y = 11$.

d. Sketch the graph of the parabola whose equation is $y = x^2 - 3x$. Display the horizontal tangent line and the tangent line found in part **c.**

47. a. Find the derivative of $f(x) = 4 - 2x^2$.

b. The graph of f has one horizontal tangent line. What is its equation?

c. At what point on the graph of f is the tangent line parallel to the line $8x + 3y = 4$?

48. Show that the function $f(x) = |x - 2|$ is not differentiable at $x = 2$.

49. Is the function $f(x) = 2|x + 1|$ differentiable at $x = 1$?

50. Let $f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$.

Does $f'(0)$ exist? *Hint*: Find the difference quotient and take the limit as $\Delta x \to 0$ from the left and from the right.

51. Let $f(x) = \begin{cases} -2x & \text{if } x < 1 \\ \sqrt{x} - 3 & \text{if } x \geq 1 \end{cases}$.

a. Sketch the graph of f.

b. Show that f is continuous but not differentiable at $x = 1$.

52. Counterexample Problem Give an example of a function that is continuous on $(-\infty, \infty)$ but is not differentiable at $x = 5$.

Estimate the derivative $f'(c)$ in Problems 53–58 by evaluating the difference quotient

$$\frac{\Delta y}{\Delta x} = \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

at a succession of numbers near c.