Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (12 pts) Sketch the region bounded between the curves $y = -x^2 - 3x + 8$ and $y = -2x - 12$. Find the area of that region.

2. Let $R$ be the region, in the first quadrant, bounded by the curves $y = x^2 + x + 1$, $y = 2x + 3$ and by the $y$-axis. Set up, but do not evaluate, an integral to compute the volume of the solid of revolution generated by revolving the region $R$ about the indicated axis of rotation:
   a. (10 pts) the $y$-axis
   b. (10 pts) the line $y = 10$

3. (12 pts) Find polar coordinates for each of the intersections points of the polar curves $r = 2\sin \theta$ and $r = 2 - 2\sin \theta$.

4. (12 pts) Consider the lemniscate given by $r^2 = 6\sin 2\theta$. Find the total area enclosed by the lemniscate.

5. (12 pts) Find the length of the curve $y = 4(x + 1)^{3/2}$ from $x = 0$ to $x = 3$.

6. (12 pts) Setup, but do not evaluate, an integral to find the surface area generated by revolving the curve $y = \frac{x^2}{2} + \frac{4}{x^2}$ from $x = 1$ to $x = 4$ about the $x$-axis.

7. (12 pts) Consider a vertical plate in tank filled with water (density $\rho = 62.4$) – see figure to the right. Calculate the fluid force against the face of the vertical plate.

8. (12 pts) Find the $y$-coordinate of the centroid of the homogenous planar region in the first quadrant bounded by the curves $y = 3 + 2x$ and $y = 19 - 6x$ and the $y$-axis.