Solution Exam II

1. $\int \frac{x}{\sqrt{9x^2-1}}\,dx = \frac{1}{18}\int u^{-\frac{1}{2}}\,du = \frac{1}{9}\sqrt{u} + c = \frac{1}{9}\sqrt{9x^2-1} + c$

$u = 9x^2-1$
$du = 18x\,dx$

2. $\int \sin^3 2x \cos^2 2x\,dx = \int (1-\cos^2 2x)\cos^2 2x \sin 2x\,dx = -\frac{1}{2}\int (1-u^2)u^2\,du$

$u = \cos 2x$
$du = -2\sin 2x\,dx$

$= -\frac{1}{2}\int (u^2-u^4)\,du = -\frac{1}{2}\left(\frac{u^3}{3}-\frac{u^5}{5}\right)+c$

$= -\frac{1}{6}\cos^3 2x + \frac{1}{10}\cos^5 2x + c$

3. $\int (x^2+1)e^{-3x}\,dx = -\frac{1}{3}(x^2+1)e^{-3x} + \int +\frac{2}{3}xe^{-3x}\,dx$

$\boxed{\begin{array}{ll} u = x^2+1 & dv = e^{-3x}\,dx \\ du = 2x\,dx & v = -\frac{1}{3}e^{-3x} \end{array}}$

$= -\frac{1}{3}(x^2+1)e^{-3x} + \frac{2}{3}\left[-\frac{1}{3}xe^{-3x} + \int +\frac{1}{3}e^{-3x}\,dx\right]$

$\boxed{\begin{array}{ll} u = x & dv = e^{-3x}\,dx \\ du = dx & v = -\frac{1}{3}e^{-3x} \end{array}}$

$= -\frac{1}{3}(x^2+1)e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + c$

4. $\int x(1+2x)^{1/3}\,dx = \frac{1}{2}\cdot\frac{1}{2}\int (u-1)u^{\frac{1}{3}}\,du = \frac{1}{4}\int (u^{4/3} - u^{1/3})\,du$

$u = 1+2x$
$du = 2dx$
$x = \frac{u-1}{2}$

$= \frac{1}{4}\left(\frac{u^{7/3}}{\frac{7}{3}} - \frac{u^{4/3}}{\frac{4}{3}}\right)+c$

$= \frac{1}{4}\left(\frac{3}{7}(1+2x)^{7/3} - \frac{3}{4}(1+2x)^{4/3}\right)+c$

5. $\displaystyle\int \frac{x}{\sqrt{4x^4+9}}dx = \frac{1}{4}\int \frac{du}{\sqrt{u^2+9}} = \frac{1}{4}\sinh^{-1}\frac{u}{3} + c$

$u = 2x^2$
$du = 4x\,dx$

$\boxed{\text{Table \# 20} \quad a = 3}$

$= \frac{1}{4}\sinh^{-1}\left(\frac{2x^2}{3}\right) + c$

6. $\displaystyle\int \tan^2 x \sec x\, dx = \int (\sec^2 x - 1)\sec x\, dx$

$\displaystyle = \int \sec^3 x\, dx - \int \sec x\, dx$

$\boxed{\text{Table \# 92} \quad n = 3 \quad a = 1}$

$\displaystyle = \frac{\sec x \tan x}{2} + \frac{1}{2}\int \sec x\, dx - \int \sec x\, dx$

$\displaystyle = \frac{\sec x \tan x}{2} - \frac{1}{2}\ln|\sec x + \tan x| + c$

7. $\displaystyle\int \frac{2-x}{\sqrt{4-x^2}}dx = \int \frac{2}{\sqrt{4-x^2}}dx - \int \frac{x}{\sqrt{4-x^2}}dx$

$u = 4 - x^2$
$du = -2x\,dx$

$\boxed{\text{Table \#28} \quad a = 2}$

$\displaystyle = 2\sin^{-1}\frac{x}{2} - \int \frac{x}{\sqrt{4-x^2}}dx$

$\displaystyle = 2\sin^{-1}\frac{x}{2} - -\frac{1}{2}\int \frac{du}{\sqrt{u}} = 2\sin^{-1}\frac{x}{2} + \sqrt{4-x^2} + c$

8. $\displaystyle\int \frac{x^2-x+3}{(x+1)^2(x^2+9)}dx = \int \frac{-\frac{1}{5}}{x+1}dx + \int \frac{\frac{1}{2}}{(x+1)^2}dx + \int \frac{\frac{1}{5}x + \frac{3}{10}}{x^2+9}dx$

$\displaystyle = -\frac{1}{5}\ln|x+1| - \frac{1}{2}\frac{1}{x+1} + \frac{1}{10}\ln(x^2+9)$

$\displaystyle \frac{x^2-x+3}{(x+1)^2(x^2+9)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+9}$

$\displaystyle + \frac{3}{10}\left(\frac{1}{3}\tan^{-1}\frac{x}{3}\right) + c$

$\boxed{\text{Table \#16} \quad a = 3}$

$$x^2 - x + 3 = A(x+1)(x^2+9) + B(x^2+9) + (Cx+D)(x+1)^2$$

@ $x=-1$ $\quad 5 = B\,10 \qquad B = \frac{1}{2}$

@ $x=0$ $\quad 3 = 9A + 9B + D$

coef x^3 $\quad 0 = A + C$

coef x^2 $\quad 1 = A + B + 2C + D$

$$A = -\frac{1}{5} \qquad C = \frac{1}{5} \qquad D = \frac{3}{10}$$

9.

a) $\frac{dy}{dx} + \frac{3}{x}y = \frac{\sin \pi x}{x^2} \qquad p(x) = \frac{3}{x} \quad q(x) = \frac{\sin \pi x}{x^2}$

$$I = e^{\int p(x)dx} = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$

$$y = \frac{1}{x^3}\left[\int x^3 \frac{\sin \pi x}{x}dx + c\right] = \frac{1}{x^3}\left[\int x \sin \pi x \, dx + c\right]$$

$\boxed{\text{Table \#78} \quad a = \pi}$

$$= \frac{1}{x^3}\left[-\frac{x}{a}\cos \pi x + \frac{1}{a^2}\sin \pi x + c\right]$$

b)

$$2 = \frac{1}{(-1)^3}\left[-\frac{(-1)}{\pi}(-1) + \frac{1}{\pi}\,0 + c\right]$$

$$-2 = \frac{-1}{\pi} + c \qquad c = \frac{1}{\pi} - 2$$

$$y = \frac{1}{x^3}\left[-\frac{x}{\pi}\cos \pi x + \frac{1}{\pi^2}\sin \pi x + \frac{1}{\pi} - 2\right]$$

10. $\displaystyle\int_0^1 \frac{dx}{(1+x)^{\frac{1}{5}}} = \frac{5}{4}(1+x)^{4/5}\Big|_0^1 = \frac{5}{4}\left[2^{4/5}-1\right]$

11. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x}} = \lim_{t\to 1^-} \int_0^t \frac{dx}{\sqrt{1-x}} = \lim_{t\to 1^-} -2\sqrt{1-x}\,\Big|_0^t$

$$= \lim_{t\to 1^-} -2\left[\sqrt{1-t} - 1\right] = 2$$

12 $\displaystyle\int_1^\infty x e^{-4x}\,dx = \lim_{N\to\infty} \int_1^N x e^{-4x}\,dx$

$$= \lim_{N\to\infty} \left[\frac{e^{-4x}}{16}(-4x-1)\right]\Big|_1^N$$

$$\boxed{\begin{array}{l}\text{Table \# 104}\\ a=-4\end{array}}$$

$$= \lim_{N\to\infty} \left[\frac{e^{-4N}}{16}(-4N-1) - \frac{e^{-4}}{16}(-5)\right]$$

$$= \frac{5}{16}e^{-4}$$