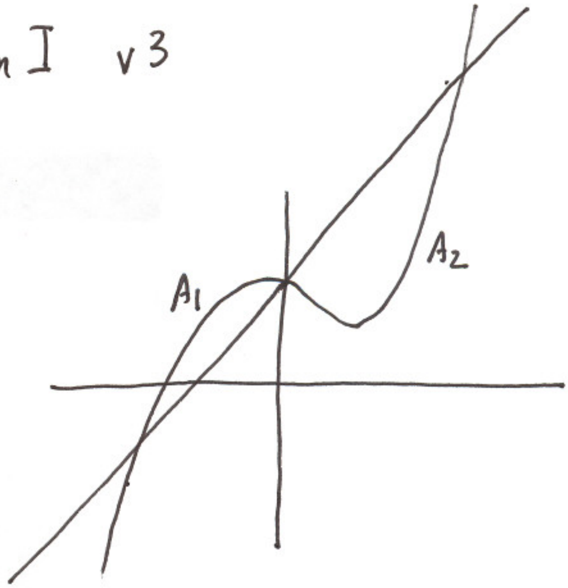
Solution Exam I v3

1.

$$x^3+x^2-4x+3 = 2x+3$$
$$x^3+x^2-6x = 0$$
$$x(x+3)(x-2) = 0$$

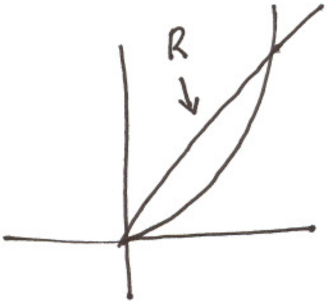


$$\text{Area} = A_1 + A_2 = \frac{253}{12}$$

$$A_1 = \int_{-3}^{0} \left[(x^3+x^2-4x+3)-(2x+3)\right]dx$$
$$= \int_{-3}^{0} (x^3+x^2-6x)\,dx = \frac{x^4}{4}+\frac{x^3}{3}-3x^2 \Big|_{-3}^{0} = \frac{63}{4}$$

$$A_2 = \int_{0}^{2} \left[(2x+3)-(x^3+x^2-4x+3)\right]dx$$
$$= \int_0^2 (-x^3-x^2+6x)\,dx = \frac{-x^4}{4}-\frac{x^3}{3}+3x^2 \Big|_0^2 = \frac{16}{3}$$

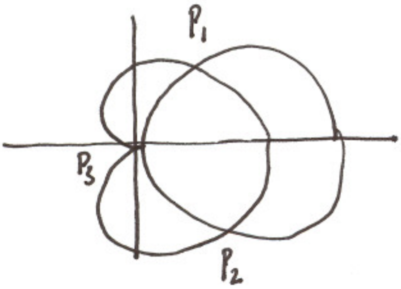
2.



a) $V = \int_0^2 2\pi x\,(2x-x^2)\,dx$

b) $V = \int_0^2 \left[\pi\,(2x+2)^2 - \pi\,(x^2+2)^2\right]dx$

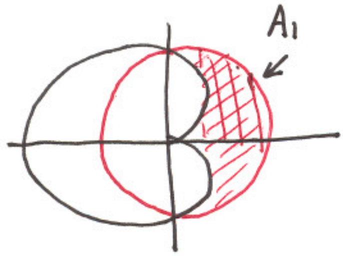
3.



$$3\cos\theta = 1+\cos\theta$$
$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}, \quad \theta = -\frac{\pi}{3}$$

$P_1 \ \left(\frac{3}{2}, \frac{\pi}{3}\right)$, $P_2 \ \left(\frac{3}{2}, -\frac{\pi}{3}\right)$, $P_3 \ (0,0)$

4.



$$\text{Area} = 2A_1 = 8-\pi$$

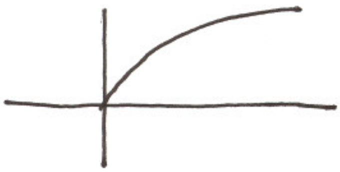
$$A_1 = \frac{1}{2}\int_0^{\pi/2} 2^2\,d\theta - \frac{1}{2}\int_0^{\pi/2} (2-2\cos$$

$$A_1 = \pi - 2\int_0^{\pi/2}(1-2\cos\theta+\cos^2\theta)\,d\theta$$

$$= \pi - 2\left[\theta - 2\sin\theta + \frac{\theta+\frac{1}{2}\sin 2\theta}{2}\right]\Bigg|_0^{\pi/2}$$

$$= \pi - \left(\pi - 4 + \frac{\pi}{2}\right) = 4-\frac{\pi}{2}$$

5.

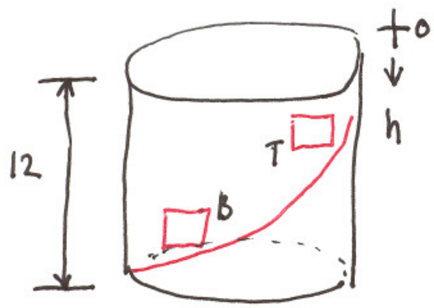


$$s = \int_0^2 \sqrt{1+(4y)^2}\,dy$$

$$x = 2y^2$$

$$\frac{dx}{dy} = 4y$$

6.



$$F_B = \int_9^{11} 62.0\,h\,3\,dh = 186\frac{h^2}{2}\Bigg|_9^{11}$$

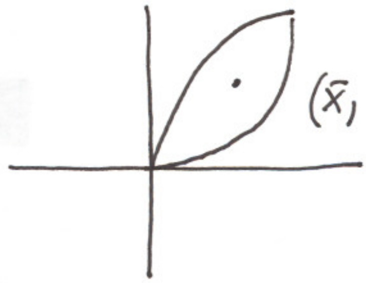
$$= 3720$$

$$F_T = \int_1^3 62.0\,h\,3\,dh = 186\frac{h^2}{2}\Bigg|_1^3$$

$$= 744$$

$$F_B - F_T = 2976$$

7.



$$m = \int_0^1 \rho \left(\sqrt[3]{x} - x^3\right) dx$$

$$= \rho \left(\frac{3}{4} x^{\frac{4}{3}} - \frac{1}{4} x^4\right)\Big|_0^1$$

$$= \rho \ \frac{1}{2}$$

$$M_y = \int_0^1 \rho \, x \left(\sqrt[3]{x} - x^3\right) dx$$

$$= \rho \left(\frac{3}{7} x^{7/3} - \frac{1}{5} x^5\right)\Big|_0^1$$

$$= \rho \ \frac{8}{35}$$

$$\bar{x} = \frac{M_y}{m} = \frac{16}{35}$$

8. $\bar{y} = \bar{x}$ by symmetry