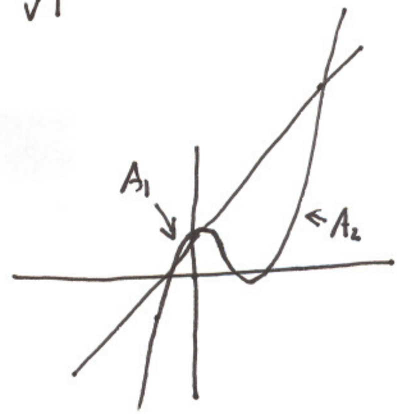
Solution Exam I v1



1. $x^3 - 3x^2 - 2x + 1 = 2x+1$

$x^3 - 3x^2 - 4x = 0$

$x(x+1)(x-4) = 0$

$\text{Area} = A_1 + A_2 = 32\frac{3}{4}$

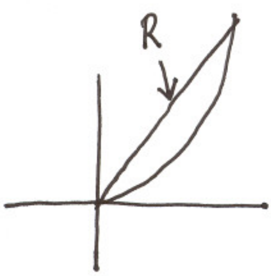
$$A_1 = \int_{-1}^{0} [(x^3 - 3x^2 - 2x + 1) - (2x+1)]\,dx$$

$$= \int_{-1}^{0} (x^3 - 3x^2 - 4x)\,dx = \frac{x^4}{4} - x^3 - 2x^2 \Big|_{-1}^{0} = \frac{3}{4}$$

$$A_2 = \int_{0}^{4} [(2x+1) - (x^3 - 3x^2 - 2x + 1)]\,dx$$

$$= \int_{0}^{4} (-x^3 + 3x^2 + 4x)\,dx = \frac{-x^4}{4} + x^3 + 2x^2 \Big|_{0}^{4} = 32$$

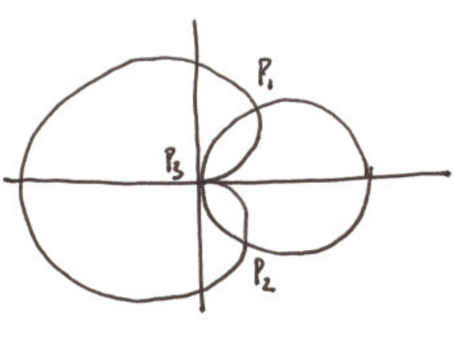
2.



a) $V = \pi \int_0^2 [(2x)^2 - (x^2)^2]\,dx$

b) $V = 2\pi \int_0^2 (x+3)[2x - x^2]\,dx$

3.



$2\cos\theta = 2 - 2\cos\theta$

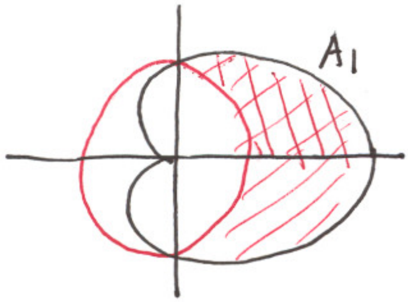
$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \ \theta = -\frac{\pi}{3}$

$P_1 \ (1, \frac{\pi}{3}), \quad P_2 \ (1, -\frac{\pi}{3}), \quad P_3 \ (0,0)$

4.



$$\text{Area} = 2A_1 = 32 + 4\pi$$

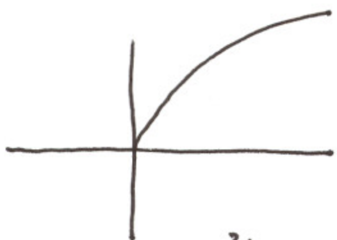
$$A_1 = \frac{1}{2}\int_0^{\frac{\pi}{2}} (4+4\cos\theta)^2\, d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}} 4^2$$

$$A_1 = 8\int_0^{\pi/2} (1 + 2\cos\theta + \cos^2\theta)\, d\theta - 4\pi$$

$$= 8\left(\theta + 2\sin\theta + \frac{\theta + \frac{1}{2}\sin 2\theta}{2}\right)\Bigg|_0^{\pi/2} - 4\pi$$

$$= 4\pi + 16 + 2\pi - 4\pi = 16 + 2\pi$$

5.



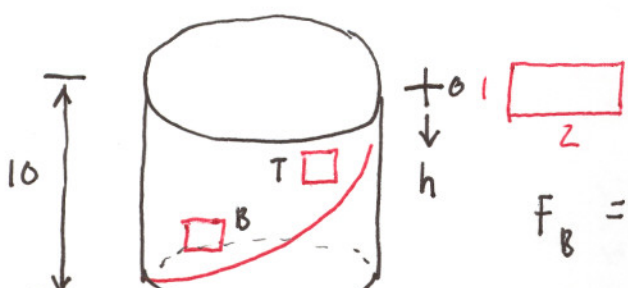
$$y = x^{3/2}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$$

$$s = \int_0^4 \sqrt{1 + \left(\tfrac{3}{2}x^{1/2}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \tfrac{9}{4}x}\, dx$$

6.



$$F_B = \int_8^9 62.0\, h\, 2\, dh = 124\frac{h^2}{2}$$

$$= 1054$$

$$F_T = \int_1^2 62.0\, h\, 2\, dh = 124\frac{h^2}{2}$$

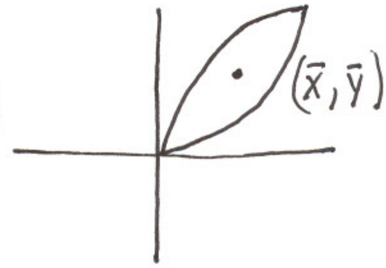
$$= 186$$

$$F_B - F_T = 868$$

7. $m = \int_0^1 \rho(\sqrt{x} - x^2)\,dx$

$= \rho\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^1$

$= \rho\,\frac{1}{3}$



$\bar{x} = \frac{M_y}{m} = \frac{9}{20}$

$M_y = \int_0^1 \rho\,x\left(\sqrt{x} - x^2\right)dx$

$= \rho\left(\frac{2}{5}x^{5/2} - \frac{1}{4}x^4\right)\Big|_0^1$

$= \rho\,\frac{3}{20}$

B. $\bar{y} = \bar{x}$ by symmetry