Review II

1. Section 7.2
   a. Integration by parts formula
   b. Repeated integration by parts
   c. Integration by parts with definite integrals
   d. Representative problems: 1, 3, 6, 8, 10, 17, 18, 24

2. Section 7.3
   a. Powers of sine and cosine $\int \sin^m x \cos^n x\, dx$
      i. One of m or n is odd
         a) $d(\cos x) = -\sin x\, dx$ or $d(\sin x) = \cos x\, dx$
         b) Pythagorean identity $\cos^2 x + \sin^2 x = 1$
      ii. Both m and n are even
         a) Reduction formula 68 or 69 on supplement
   b. Powers of tangent and secant $\int \tan^m x \sec^n x\, dx$
      i. n is even
         c) $d(\tan x) = \sec^2 x\, dx$
         b) Pythagorean identity $\tan^2 x + 1\, x = \sec^2 x$
      ii. m is odd
         a) $d(\sec x) = \sec x \tan x\, dx$
         b) Pythagorean identity $\tan^2 x = \sec^2 x - 1$
      iii. m is even and n is odd
         a) Pythagorean identity $\tan^2 x = \sec^2 x - 1$
         b) Reduction formula 92 on supplement
   c. Trigonometric substitution
      i. Form $\sqrt{a^2 - x^2}$ --> substitute $x = a \sin \theta$
      ii. Form $\sqrt{a^2 + x^2}$ --> substitute $x = a \tan \theta$
      iii. Form $\sqrt{x^2 - a^2}$ --> substitute $x = a \sec \theta$
      iv. Form $\sqrt{a x^2 + b x + c}$ --> complete square and convert to above form
   d. Representative problems: 3, 7, 10, 13, 17, 18, 21, 20, 24, 27, 35, 38, 39, 41, 47, 48

3. Section 7.4
   a. Proper rational function
   b. Denominator factored into product of linear factors or irreducible quadratic factors
   c. Cases
      i. Non-repeated linear factors
      ii. Repeated linear factors
      iii. Non-repeated irreducible quadratic factors
      iv. Repeated irreducible quadratic factors
   d. Convert partial fraction equation to polynomial equation by multiplying by common denominator
   e. Solve for undetermined constants
      i. Compare coefficients
      ii. Compare functional values
      iii. Heavy-side covering method (for non-repeated linear factors)
   f. Integrate the partial fraction summands
   g. Rational functions of sine and cosine
      i. Weierstrass substitution
         a) $z = \tan x/2 \qquad dx = \frac{2}{1 + z^2}\, dz$
         b) $\sin x = \frac{2z}{1 + z^2} \qquad \cos x = \frac{1 - z^2}{1 + z^2}$
   h. Representative problems: 13, 14, 18, 19, 20, 22, 25, 28, 30, 33, 36, 42, 44, 49, 50

4. Section 7.5
   a. Summary of integration techniques
   b. Table 7.2: Integration Strategy

5. Section 7.6
   a. First order separable differential equations
   b. First order linear differential equations
      i. General or standard form $\frac{dy}{dx} + P(x)\,y = Q(x)$
      ii. General solution
         a) Integrating factor $I(x) = e^{\int P(x)\,dx}$
         b) Formula $y = \frac{1}{I(x)}\left[\int Q(x)\,I(x) + C\right]$
   c. Initial Value Problems (IVP)
      i. $\frac{dy}{dx} + P(x)\,y = Q(x)\ ,\ y(x_0) = y_0$
   d. Population growth modeling
      i. Exponential growth $\frac{dP}{dt} = k\,P$
      ii. Logistic growth $\frac{dP}{dt} = k\,P\,(B - P)$
   e. Modeling dilution = Inflow rate (S) − Outflow rat
   f. Representative problems: 1, 3, 5, 8, 11, 12, 13, 19, 22, 23, 34

6. Section 7.7
   a. Improper integrals with infinite limits of integration
      $\int_a^{\infty} f(x)\,dx = \lim_{N \to \infty} \int_a^N f(x)\,dx$
      i. Terminology: Convergence vs Divergence
      ii. p integrals $\int_1^{\infty} \frac{1}{x^p}\,dx$
   b. Improper integrals with unbounded integrands (at a or b)
      $\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$
      $\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$
      i. Terminology: Convergence vs Divergence
   c. Representative problems: 3, 4, 6, 9, 12, 14, 15, 20, 22, 27, 31, 34, 35, 39, 40

7. Section 7.8
   a. Hyperbolic trigonometric functions
      i. Definitions
      ii. Domain, Range
      iii. Graphs
      iv. Restrictions for monotonicity
      v. Functional relationships, identities
      vi. Rules for differentiation
      vii. Integration
      viii. Parallels to Circular Trigonometric Functions
   b. Inverse trigonometric functions
      i. Definitions
      ii. Domain, Range
      iii. Graphs
      iv. Restrictions for monotonicity
      v. Functional relationships, identities
      vi. Rules for differentiation
      vii. Integration
      viii. Parallels to Circular Trigonometric Functions
   c. Representative problems: 13, 14, 16, 17, 18, 20, 22, 26, 28, 30, 32, 34, 37, 38, 41, 43