MATH 1352-H02 Exam III April 16, 2002

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

1. Determine which sequences $\{a_n\}$ converge and which diverge. Find the limit of each convergent sequence. In each case, provide reasons for your conclusion. Do **three (3)** of the following problems

a. $a_n = 1 + \dfrac{(-1)^n}{n}$

b. $a_n = \dfrac{\sqrt{n-1}}{\sqrt{n}}$

c. $a_n = \dfrac{n^2 - 2n - 1}{n + 1}$

d. $a_n = \dfrac{2n + \sin n}{n - \cos 5n}$

2. Find the sum of the following infinite series, if it converges. Provide reasons for your conclusion.

a. $\displaystyle\sum_{n=0}^{\infty} \frac{5}{2^n} - \left(\frac{3}{4}\right)^n$

3. Determine which of the following infinite series converge and which diverge. In each case, provide reasons for your conclusion. Do **four (4)** of the following problems

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{10n + 1}{n(n+1)(2n-1)}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

e. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

4. Determine which of the following infinite series converge absolutely, which converge conditionally and which diverge. In each case, provide reasons for your conclusion. Do **two (2)** of the following problems

a. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+11}{2n+7}$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{(n+2)^2}$

c. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+1}{2^n}$

5. It can be shown, using powers series, that

$$\sin \frac{\pi}{12} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{\pi}{12}\right)^{2n+1} = \frac{\pi}{12} - \frac{1}{3!}\left(\frac{\pi}{12}\right)^3 + \frac{1}{5!}\left(\frac{\pi}{12}\right)^5 - \frac{1}{7!}\left(\frac{\pi}{12}\right)^7 + \cdots.$$

Use the above series to find the value of $\sin \frac{\pi}{12}$ to six decimal places accuracies. Provide reasons for your conclusion.