

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

1. [16 pts.] Find the derivative of  $y$  with respect to  $x$  or  $t$  as appropriate. Simplify where appropriate.

a.  $y = \tanh^{-1}(\sin x)$

b.  $y = \ln(2e^{-t^2} \cos t)$

2. [16 pts.] Find each of the following limits, if the limits exist.

a.  $\lim_{t \rightarrow 0} \frac{e^t - 1 - t}{t^2}$

b.  $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{\sin x}$

In problems 3-4, write down an integral (or integrals) to represent the solution of the stated physical problem. Do **NOT** evaluate the integral (or integrals).

In problems 3-4, let  $f(x) = \frac{4}{3}x^3 - 2x^2 + \frac{2}{3}x$  and  $g(x) = \sin \pi x$ . Note, that the graphs of  $f$  and  $g$  have a common intersection point at  $x = -1/2$ . Also, note that  $f(x) = \frac{2}{3}x(2x-1)(x-1)$ .

3. [10 pts.] Find the area of the bounded region enclosed by the graphs of  $f$  and  $g$ .
4. [16 pts.] Let  $G$  the bounded region in the first quadrant which is bounded above by the graph of  $f$  and below by the  $x$ -axis. Find the volume of the solid of revolution determined by revolving  $G$  about

a. the line  $y = -1$

b. the line  $x = -1$

5. [40 pts.] Evaluate five (5) of the following integrals:

a.  $\int \frac{dv}{v(3 - \ln v)}$

b.  $\int \frac{2 dt}{\sqrt{5 - 9t^2}}$

c.  $\int x^2 \sqrt{x+6} dx$

d.  $\int \frac{x^4 - x + 2}{x^2(x+2)} dx$

e.  $\int_3^5 \frac{x}{\sqrt{x^2 - 9}} dx$

f.  $\int \frac{\sqrt{3 - v^2}}{v^2} dv$

6. [16 pts.] Consider the following functions  $f$  and  $g$ . Determine whether  $f$  grows faster than  $g$ , at the same rate as  $g$ , or slower than  $g$ .

a.  $f(x) = 2x + 1, g(x) = x^2 + 1$

b.  $f(x) = \ln(2x + 1), g(x) = \ln(x^2 + 1)$

7. [20 pts.] Determine whether the following sequences  $\{a_n\}$  converge or diverge. If they converge, determine the limit.

a.  $a_n = \frac{\ln n}{\ln(n^2 + 1)}$

b.  $a_n = \left(\frac{n}{n+1}\right)^n$

8. [30 pts.] Determine whether the following series converge absolutely, converge conditionally or diverge.

a.  $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{6n^3 + 1}$

b.  $\sum_{n=0}^{\infty} (-1)^n n \sin \frac{1}{n}$

c.  $\sum_{n=1}^{\infty} \frac{n! \ln n}{(n+2)!}$

9. [24 pts.] Consider the power series  $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n+1)3^n}$ . Determine the radius of convergence and the interval of convergence of the power series. For which values of  $x$  does the power series converge absolutely? converge conditionally?

10. [10 pts.] Find the first four (4) nonzero terms of the Taylor series centered at  $x = 0$  of the function  $f(x) = \sqrt{1+x}$ .

11. [10 pts.] Consider the function  $f(x) = e^{-x^2}$ . If  $f$  is approximated on the interval  $[-0.3, 0.3]$  by  $p(x) = 1 - x^2 + \frac{x^4}{2}$ , what estimate of the error can be made?