Section 5.8

I. Integration $\int_a^b f(x)\,dx$

A. If possible – Apply the Fundamental Theorem of Calculus

B. If not possible – Numerical Approximation

1. Partition $[a,b]$ in n subintervals

a. $\Delta x = \dfrac{b-a}{n}$

b. $x_k = a + k\Delta x$

2. Approximation by Rectangles

a. Right-hand Rule $I \approx \sum_{k=1}^{n} f(x_k)\Delta x$

b. Mid-point Rule

(1) $x_k^* = \dfrac{x_k + x_{k-1}}{2} = a + (k - \tfrac{1}{2})\Delta x$

(2) $I \approx \sum_{k=1}^{n} f(x_k^m)\Delta x$

3. Trapezoid Rule

a. Area Element $A_k = \frac{1}{2}\left[f(x_{k-1}) + f(x_k)\right]\Delta x$

b. $I \approx \left[\frac{1}{2} f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \frac{1}{2} f(x_n)\right]\Delta x$

4. Simpson's Rule

a. n even

b. Area Element $A_{2k} = \frac{1}{3}\left[f(x_{2k-2}) + 4f(x_{2k-1}) + f(x_{2k})\right]\Delta x$

c.

$$I \approx \tfrac{1}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right]\Delta x$$

II. Error Estimates

A. Mid-point Rule $|E_n| \le \frac{(b-a)^3}{24n^2} M$ where $M = \max_{x \in [a,b]} |f''(x)|$

B. Trapezoid Rule $|E_n| \le \frac{(b-a)^3}{12n^2} M$ where $M = \max_{x \in [a,b]} |f''(x)|$

C. Simpson's Rule $|E_n| \le \frac{(b-a)^5}{180n^4} K$ where $K = \max_{x \in [a,b]} |f^{(iv)}(x)|$