Section 4.4 (cont.)

I. Infinite Limits

a. $\lim_{x \to c} f(x) = +\infty$ ( $\lim_{x \to c^+} f(x) = +\infty$ , $\lim_{x \to c^-} f(x) = +\infty$ )

b. $\lim_{x \to c} f(x) = -\infty$ ( $\lim_{x \to c^+} f(x) = -\infty$ , $\lim_{x \to c^-} f(x) = -\infty$ )

Examples

$f(x) = \dfrac{x+1}{x-2}, \ c = 2$ $\qquad$ $f(x) = \dfrac{x^2 + x}{x^2 - 2x + 1}, \ c = 1$

$f(x) = (x^2 + 1)\tan x, \ x = \dfrac{\pi}{2}$ $\qquad$ $f(x) = \ln x, \ c = 0$

II. Vertical Asymptote

III. Sketching the Graph of a Function $y = f(x)$ with (potentially) Asymptotes

a. Identify the domain

b. Identify any intercepts

c. Identify any asympototes

1. Horizontal asymptotes ( $y = L$ where $L = \lim_{x \to \pm\infty} f(x)$ )
2. Vertical asympototes ( $x = c$ where $\lim_{x \to c^{\pm}} f(x) = \pm\infty$ )

d. Identify the critical numbers: $f'(x)$ vanishes

e. Identify the intervals of monotonicity:

1. Intervals where $f'(x) > 0 \Leftrightarrow$ Intervals where $f$ is strictly increasing
2. Intervals where $f'(x) < 0 \Leftrightarrow$ Intervals where $f$ is strictly decreasing

f. Classify each critical point as a relative maximum, relative minimum or neither

g. Identify the second-order critical numbers: $f''(x)$ vanishes

h. Identify the intervals of concavity

   a. Intervals where $f''(x) > 0 \Leftrightarrow$ Intervals where $f$ is concave up
   b. Intervals where $f''(x) < 0 \Leftrightarrow$ Intervals where $f$ is concave down

i. Identify any inflection points

j. Construct a table with a few specific values of the function

k. Sketch the graph of $y = f(x)$ incorporating all of the above information