Section 4.4 (cont.)

I. Infinite Limits

a. \( \lim_{x \to c} f(x) = +\infty \) ( \( \lim_{x \to c^+} f(x) = +\infty \), \( \lim_{x \to c^-} f(x) = +\infty \) )

b. \( \lim_{x \to c} f(x) = -\infty \) ( \( \lim_{x \to c^+} f(x) = -\infty \), \( \lim_{x \to c^-} f(x) = -\infty \) )

Examples

\[
\begin{align*}
f(x) &= \frac{x+1}{x-2}, \quad c = 2 \\
f(x) &= \frac{x^2 + x}{x^2 - 2x + 1}, \quad c = 1 \\
f(x) &= (x^2 + 1) \tan x, \quad x = \frac{\pi}{2} \\
f(x) &= \ln x, \quad c = 0
\end{align*}
\]

II. Vertical Asymptote

III. Sketching the Graph of a Function \( y = f(x) \) with (potentially) Asymptotes

a. Identify the domain

b. Identify any intercepts

c. Identify any asymptotes

1. Horizontal asymptotes ( \( y = L \) where \( L = \lim_{x \to \pm\infty} f(x) \) )

2. Vertical asymptotes ( \( x = c \) where \( \lim_{x \to c^\pm} f(x) = \pm\infty \) )

d. Identify the critical numbers: \( f'(x) \) vanishes

e. Identify the intervals of monotonicity:

1. Intervals where \( f'(x) > 0 \) \( \iff \) Intervals where \( f \) is strictly increasing

2. Intervals where \( f'(x) < 0 \) \( \iff \) Intervals where \( f \) is strictly decreasing

f. Classify each critical point as a relative maximum, relative minimum or neither
g. Identify the second-order critical numbers: \( f''(x) \) vanishes

h. Identify the intervals of concavity
   a. Intervals where \( f''(x) > 0 \) \iff\ Intervals where \( f \) is concave up
   b. Intervals where \( f''(x) < 0 \) \iff\ Intervals where \( f \) is concave down

i. Identify any inflection points

j. Construct a table with a few specific values of the function

k. Sketch the graph of \( y = f(x) \) incorporating all of the above information