

Section 4.4

I. Limits at Infinity $\lim_{x \rightarrow \infty} f(x) = L$

- a. Informal Sense: The computed values of $y = f(x)$ can be made arbitrarily close to the value L by choosing x sufficiently large
- b. Graphical Sense: Given any $(\varepsilon -)$ band about the horizontal line $y = L$ the graph of $y = f(x)$ can be made to lie inside that $(\varepsilon -)$ band by choosing x sufficiently large
- c. Formal Sense: For each $\varepsilon > 0$ there exists a number N such that $|f(x) - L| < \varepsilon$ whenever $x > N$

II. Limit Rules

- | | |
|---------------------------|-------------------------|
| a. Constant Rule | b. Reciprocal Rule |
| c. Constant Multiple Rule | d. Sum Rule |
| e. Difference Rule | f. Product Rule |
| g. Quotient Rule | h. Algebraic Power Rule |

III. Special Limits A.

- | | |
|---|--|
| a. $\lim_{x \rightarrow \infty} x^{-1} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$ | b. $\lim_{x \rightarrow -\infty} x^{-1} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$ |
| c. $\lim_{x \rightarrow \infty} e^{-x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$ | d. $\lim_{x \rightarrow \infty} \frac{1}{\ln x} = 0$ |
| e. $\lim_{x \rightarrow \infty} x^n e^{-x} = 0, \quad n > 0$ | f. $\lim_{x \rightarrow \infty} \frac{\ln x}{x^n} = 0, \quad n > 0$ |

IV. Special Limits B.

a. $\lim_{x \rightarrow \infty} x = \infty$

b.

c. $\lim_{x \rightarrow \infty} e^x = \infty$

d. $\lim_{x \rightarrow \infty} \ln x = \infty$

V. Evaluating Limits at Infinity

a. Rational Functions

$$\lim_{x \rightarrow \infty} \frac{3x+4}{5-2x}$$

$$\lim_{x \rightarrow -\infty} \frac{3x+4}{5-2x}$$

$$\lim_{x \rightarrow \infty} \frac{28x}{x^2-15x-12}$$

$$\lim_{x \rightarrow \infty} \frac{4x^5-3x^2+2x-8}{10x^5-3x^4+5x-96}$$

$$\lim_{x \rightarrow -\infty} \frac{2x^6-5x^3+4}{74x^4+80x^3+19x^2+108x+256}$$

b. Others

$$\lim_{x \rightarrow \infty} \frac{\sin 4x}{e^x}$$

$$\lim_{x \rightarrow \infty} \frac{x \ln x}{e^x}$$

$$\lim_{x \rightarrow \infty} \sqrt{\frac{4x^2-1}{2x^2+x+4}}$$

$$\lim_{x \rightarrow \infty} \frac{x^\pi + x^3}{x^{22/7}}$$