Section 4.1

I. Extreme Values of a Function $y = f(x)$ on an Interval I

a. $f(x_M)$ is an absolute maximum of $y = f(x)$ on I if $f(x_M) \geq f(x)$ for all $x \in I$

b. $f(x_m)$ is an absolute minimum of $y = f(x)$ on I if $f(x_m) \leq f(x)$ for all $x \in I$

c. Note the terminology difference between the location of the extrema (x_M or x_m) and the value of the extrema ($f(x_M)$ or $f(x_m)$)

II. **Extreme Value Theorem** *Let $y = f(x)$ be continuous on the closed bounded interval $[a,b]$. Then, $y = f(x)$ has an absolute maximum and an absolute minimum on $[a,b]$. I.e., there exists points x_M , $x_m \in [a,b]$ such that $f(x_M)$ is an absolute maximum and $f(x_m)$ is an absolute minimum of $y = f(x)$ on $[a,b]$.*

a. Requirements for application of the Extreme Value Theorem

1. The interval must be a closed bounded interval
2. The function $y = f(x)$ must be continuous on $[a,b]$.

III. Relative Extreme Values of a Function $y = f(x)$

a. $f(x_M)$ is an relative maximum of $y = f(x)$ if $f(x_M) \geq f(x)$ for all x near x_M [in some open interval around x_M]

b. $f(x_m)$ is an relative minimum of $y = f(x)$ if $f(x_m) \leq f(x)$ for all x near x_m [in some open interval around x_m]

Examples

IV. Critical Numbers: If $y = f(x)$ is defined at $x = c$ and either (i) $f'(c) = 0$ or (ii) $f'(c)$ does not exist (Language: $y' = f'(x)$ vanishes at $x = c$), then we say that the number $x = c$ is a critical number of $y = f(x)$. The point $(c, f(c))$ on the graph of $y = f(x)$ is a called a critical point of the graph of $y = f(x)$.

a. Note the terminology difference between a critical number c of the function f and a critical point $(c, f(c))$ of the graph of f

Examples

V. **Critical Number Theorem** *Let $y = f(x)$ be continuous on an interval I. If $f(c)$ is a relative extrema value of $y = f(x)$ on I, then c is a critical number of $y = f(x)$.*

VI. Algorithm for Finding Absolute Extreme Values

Examples

VII. Applied Problem – Maximum Value Problem

Examples