Section 3.8

I. Tangent Line Approximation

a. Let $y = f(x)$ be a function which is differentiable at $x = a$. Then, the tangent line to the graph of $y = f(x)$ at the point $P = (a, f(a))$ has slope $m = f'(a)$ and equation $y - f(a) = f'(a)(x - a)$ or alternately $y = f(a) + f'(a)(x - a)$.

b. For x_1 "near" *a* the value $f(x_1)$ is reasonably approximated by the *y*-value on the tangent line, i.e., $f(x_1) \approx f(a) + f'(a)(x_1 - a)$.

c. We call $L(x) = f(a) + f'(a)(x - a)$ the *linear approximation* to f at $x = a$ or alternately, the *linearization* of f at $x = a$.

d. Graphical Interpretation

Examples

II. Differentials

a. Let $y = f(x)$ be a function which is differentiable at $x = a$. Then, the differential of y (or f) is $dy = f'(x)dx$ (or $df = f'(x)dx$)

b. Graphical Interpretation

c. Differential Rules

Examples

III. Error Approximations: $f(x)$ vs $f(x + \Delta x)$

a. Error: $\Delta f = f(x+\Delta x) - f(x) \approx f'(x)\Delta x = df$

b. Relative Error: $\dfrac{\Delta f}{f} \approx \dfrac{df}{f}$

Examples

IV. Newton's Method for Root Approximation

a. Let $y = f(x)$ be a function which has a root at x^*. Given a reasonable initial "guess" x_n as to the value of the root x^*, then $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ will be a better approximation for the value of x^*.

b. Graphical interpretation

c. Algorithm (Page 172)

Examples