Section 3.8

I. Tangent Line Approximation

a. Let \( y = f(x) \) be a function which is differentiable at \( x = a \). Then, the tangent line to the graph of \( y = f(x) \) at the point \( P = (a, f(a)) \) has slope \( m = f'(a) \) and equation \( y - f(a) = f'(a)(x - a) \) or alternately \( y = f(a) + f'(a)(x - a) \).

b. For \( x_1 \) “near” \( a \) the value \( f(x_1) \) is reasonably approximated by the \( y \)-value on the tangent line, i.e., \( f(x_1) \approx f(a) + f'(a)(x_1 - a) \).

c. We call \( L(x) = f(a) + f'(a)(x - a) \) the linear approximation to \( f \) at \( x = a \) or alternately, the linearization of \( f \) at \( x = a \).

d. Graphical Interpretation

Examples

II. Differentials

a. Let \( y = f(x) \) be a function which is differentiable at \( x = a \). Then, the differential of \( y \) (or \( f \)) is \( dy = f'(x)dx \) (or \( df = f'(x)dx \)).

b. Graphical Interpretation

c. Differential Rules

Examples

III. Error Approximations: \( f(x) \) vs \( f(x + \Delta x) \)
a. Error: $\Delta f = f(x + \Delta x) - f(x) \approx f'(x)\Delta x = df$

b. Relative Error: $\frac{\Delta f}{f} \approx \frac{df}{f}$

Examples

IV. Newton’s Method for Root Approximation

a. Let $y = f(x)$ be a function which has a root at $x^*$. Given a reasonable initial “guess” $x_n$ as to the value of the root $x^*$, then $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ will be a better approximation for the value of $x^*$.

b. Graphical interpretation

c. Algorithm (Page 172)

Examples