Section 3.1

I. Secant Line through \( (x_0, f(x_0)) \) to \( (x_0 + \Delta x, f(x_0 + \Delta x)) \)

Secant Line through \( (x_0, f(x_0)) \) to \( (x_1, f(x_1)) \)

a. \[
\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}
\]

b. Difference Quotient
c. Slope of Secant line
d. Average Rate of Change of \( f \) over the interval \([x_0, x_0 + \Delta x]\)

II. Tangent Line = Limiting Case of Secant Line through \( (x_0, f(x_0)) \) to \( (x_0 + \Delta x, f(x_0 + \Delta x)) \)

Tangent Line = Limiting Case of Secant Line through \( (x_0, f(x_0)) \) to \( (x_1, f(x_1)) \)

a. \[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = f'(x_0)
\]

b. Derivative = \( \lim_{\Delta x \to 0} \) Difference Quotient
c. Slope of Tangent Line = \( \lim_{\Delta x \to 0} \) Slope of Secant Line
d. Instantaneous Rate of Change of \( f \) at \( x_0 \) = \( \lim_{\Delta x \to 0} \) Average Rate of Change of \( f \) over the interval \([x_0, x_0 + \Delta x]\)

Examples

III. Definition of Derivative of \( f \) at \( x \)

Examples
IV. Equation of Tangent Line to a curve \( y = f(x) \) at \( x = x_0 \)

Equation of Normal Line to a curve \( y = f(x) \) at \( x = x_0 \)

Examples

V. Relationship between \( f \) and \( f' \)

VI. Existence of Derivative

VII. Differentiability and Continuity