Lecture Summaries

**10 Jan 1.1**

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|---|---|
| Critical Ideas | distance on a number line, absolute value, distance in a plane, trigonometry, solving trigonometric equations |
| Terms/Definitions | real numbers, real number line, absolute value, distance, interval notation, bounded interval, open interval, half-open interval, closed interval, absolute value equation property, tolerance, absolute error, horizontal change, vertical change, midpoint, analytical geometry, graph of an equation, unit circle, completing the square, degree, radian |
| Facts/Rules/Theorems | order properties (tricotomy law, transitive law of inequality, additive law of inequality, multiplicative law of inequality), absolute value formula, distance formula on real number line, properties of absolute value, intervals (inequality notation, interval notation, graphical representation), **theorem**: distance formula in the plane, midpoint formula, standard form for the equation of a circle |
| Supplementary Problems | **1.1**: every other odd 1-45 |

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**12 Jan 1.2**

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|---|---|
| Critical Ideas | slope of a line, forms for the equation of a line, parallel and perpendicular lines |
| Terms/Definitions | inclination, slope, angle of inclination, parallel, perpendicular |
| Facts/Rules/Theorems | formula for the slope of a line, formula for the angle of inclination of a line, forms of the equation of a line (standard form, slope-intercept form, point-slope form, two-intercept form, horizontal line, vertical line), slope criteria for parallel and perpendicular lines |
| Supplementary Problems | **1.2**: every other odd 1-45 |

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**17 Jan 1.3**

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|---|---|
| Critical Ideas | definition of a function, functional notation, domain of a function, composition of functions, graph of a function, classification of functions |
| Terms/Definitions | function, image, domain, range, onto function, one-to-one function, bounded function, variables, dependent variable, independent variable, evaluate, difference quotient, piecewise-defined function, domain convention, undefined, equal functions, hole, composite function, graph, vertical line test, $y$-intercept, $x$-intercept, symmetry, symmetric with respect to the y-axis, even function, symmetric with respect to the origin, odd function, polynomial function, degree, leading coefficient, constant term, constant function, linear function, quadratic function, cubic function, quartic function, rational function, power function, algebraic function, transcendental function, trigonometric functions, exponential functions, logarithmic functions |

| Facts/Rules/Theorems | rule for equality of two functions, rules for finding the *y*-intercepts and *x*-intercept(s) of a function, test for y-axis symmetry of the graph of a function, test for origin symmetry of the graph of a function |
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| Supplementary Problems | **1.3**: every other odd 1-61 |

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19 Jan **1.4**

| Critical Ideas | inverse functions, criteria for existence of an inverse $f^{-1}$, graph of $f^{-1}$, inverse trigonometric functions, inverse trigonometric identities |
|---|---|
| Terms/Definitions | inverse of $f$, one-to-one function, horizontal line test, strictly increasing, strictly decreasing, strictly monotonic, reference triangle |
| Facts/Rules/Theorems | **theorem**: a strictly monotonic function has an inverse, procedure for finding the graph of the inverse of a function, graphs of $\sin^{-1} x$, $\tan^{-1} x$, inversion formulas for trigonometric functions |
| Supplementary Problems | **1.4**: every other odd 5-41 |

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22 Jan **2.1**

| Critical Ideas | informal computation of limits, one-sided limits, limits that do not exist, formal definition of a limit |
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| Terms/Definitions | limit of a function, right-hand limit, left-hand limit, diverge, tend to infinity, divergence by oscillation, epsilon-delta definition |
| Facts/Rules/Theorems | $\lim\limits_{x \to c} f(x) = L$, $\lim\limits_{x \to c^+} f(x) = L$, $\lim\limits_{x \to c^-} f(x) = L$, **theorem**: one-side limit theorem, $\lim\limits_{x \to c} f(x) = +\infty$, $\lim\limits_{x \to c} f(x) = -\infty$ |
| Supplementary Problems | **2.1**: every other odd 1-41 |

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24 Jan **2.2**

| Critical Ideas | computations with limits, using algebra to find limits, limits of piecewise-defined functions, two special trigonometric limits |
|---|---|
| Terms/Definitions | squeeze rule |
| Facts/Rules/Theorems | basic properties and rules for limits (constant rule, limit of $x$ rule, multiple rule, sum rule, difference rule, product rule, quotient rule, power rule), **theorem**: limit of a polynomial function, **theorem**: limit of a rational function (where defined), **theorem**: limits of trigonometric functions (where defined), **theorem**: special limits ( $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$, $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x} = 0$ ) |
| Supplementary Problems | **2.2**: every other odd 1-57 |

26 Jan **2.3**

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|---|---|
| Critical Ideas | intuitive notion of continuity, definition of continuity, continuity theorems, continuity on an interval, the intermediate value theorem |
| Terms/Definitions | continuous at a point $x=c$, discontinuity, continuous from the right at $a$, continuous from the left at $a$, continuous on the open interval $(a,b)$, continuous on the half-open interval $[a,b)$, continuous on the half-open interval $(a,b]$, continuous on the closed interval $[a,b]$, suspicious point, intermediate value property, root |
| Facts/Rules/Theorems | **theorem**: continuity theorem (polynomials, rational functions, trigonometric functions, inverse trigonometric functions are continuous (where defined)), **theorem**: properties of continuous functions (scalar multiples, sums and differences, products, quotients (where defined), compositions (where defined) of continuous functions are again continuous functions), **theorem**: intermediate value theorem, **theorem**: root location theorem |
| Supplementary Problems | **2.3**: every other odd 1-41 |

29 Jan **2.4**

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|---|---|
| Critical Ideas | exponential functions, logarithmic functions, natural exponential and logarithmic functions, continuous compounding of interest |
| Terms/Definitions | completeness property, exponential function with base $b$, logarithm of $x$ to the base $b$, exponent to the base $b$, natural exponential base, natural exponential function, natural logarithm, common logarithm, continuous compounding of interest, present value, principal, interest rate, future value |
| Facts/Rules/Theorems | **theorem**: properties of exponential functions (equality rule, inequality rules, product rule, quotient rule, power rules), **theorem**: properties of logarithmic functions (equality rule, inequality rules, product rule, quotient rule, power rule, inversion rules, special values), **theorem**: basic properties of natural logarithm ($\ln 1 = 0$, $\ln e = 1$, $e^{\ln x} = x$, $\ln e^y = y$, $b^x = e^{x \ln b}$), **theorem**: change of base ($\log_b x = \dfrac{\ln x}{\ln b}$) |
| Supplementary Problems | **2.4**: every other odd 1-61 |

02 Feb **3.1**

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|---|---|
| Critical Ideas | tangent lines, the derivative, relationship between the graphs of $f$ and $f'$, existence of derivatives, continuity and differentiability, derivative notation |
| Terms/Definitions | secant line, slope of tangent line, difference quotient, derivative of $f$, differentiate $f$ at $x$, $f$ differentiable at $x$ |
| Facts/Rules/Theorems | formula for the slope of a tangent line to $y = f(x)$ at $x = x_0$, formula for the |

derivative of a function $f$ at $x$ ( $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x)$ ), **theorem**: formula for the equation of a tangent line to $y = f(x)$ at $x = x_0$, **theorem**: differentiability implies continuity,

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| Supplementary Problems | **3.1**: every other odd 5-61 |

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## 05 Feb **3.2**

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| Critical Ideas | derivative of a constant function, derivative of a power function, procedural rules for finding derivatives, higher-order derivatives |
| Terms/Definitions | first derivative of $f$, second derivative of $f$, third derivative of $f$, $n$th derivative |
| Facts/Rules/Theorems | **theorem**: constant rule, **theorem**: power rule, **theorem**: basic procedural rules (constant multiple, sum rule, difference rule, linearity rule, product rule, quotient rule) |
| Supplementary Problems | **3.2**: every other odd 1-49 |

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## 09 Feb **3.3**

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| Critical Ideas | derivatives of the sine and cosine functions, differentiation of the other trigonometric functions, derivatives of exponential and logarithmic functions |
| Terms/Definitions | |
| Facts/Rules/Theorems | **theorem**: trigonometric functions ( $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$ ), **theorem**: other trigonometric functions, **theorem**: natural exponential function ( $(e^x)' = e^x$ ), **theorem**: natural logarithm function ( $(\ln x)' = \dfrac{1}{x}$ ) |
| Supplementary Problems | **3.3**: every other odd 1-53 |

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## 12 Feb **3.4**

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| Critical Ideas | average and instantaneous rate of change, introduction to mathematical modeling, rectilinear motion (modeling in physics), falling body problem |
| Terms/Definitions | average rate of change of $y$ with respect to $x$, instantaneous rate of change, relative rate of change, mathematical modeling, abstraction, velocity, acceleration, speed, advancing, retreating, accelerating, decelerating, position, falling body problem |
| Facts/Rules/Theorems | |
| Supplementary Problems | **3.4**: every other odd 5-61 |

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## 14 Feb **3.5**

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| Critical Ideas | introduction to the chain rule, extended derivative formulas, justification of the chain rule |
| Terms/Definitions | horizontal tangent line |
| Facts/Rules/Theorems | **theorem**: chain rule ( $[f(g(x))]' = f'(g(x))g'(x)$ ), extended power rule ( $[u^n]' = nu^{n-1}u'$ ), extended trigonometric rules, extended exponential and logarithmic rules |
| Supplementary Problems | **3.5**: every other odd 5-61 |

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**16-19 Feb 3.6**

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| Critical Ideas | general procedure for implicit differentiation, derivative formulas for the inverse trigonometric functions, logarithmic differentiation |
| Terms/Definitions | explicitly defined function, implicitly defined function, implicit differentiation, logarithmic differentiation |
| Facts/Rules/Theorems | **theorem**: differentiation rules for inverse trigonometric functions, **theorem**: differentiation of exponential and logarithmic functions with base $b$ |
| Supplementary Problems | **3.6**: every other odd 1-57 |

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**21 Feb 3.7**

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| Critical Ideas | |
| Terms/Definitions | related rate problems, general situation, specific situation |
| Facts/Rules/Theorems | |
| Supplementary Problems | **3.7**: every other odd 1-45 |

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**23 Feb 3.8**

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|---|---|
| Critical Ideas | tangent line approximation, the differential, error propagation, marginal analysis in economics, the Newton-Raphson method for approximating roots |
| Terms/Definitions | linear approximation, linearization, incremental approximation formula, differential of $x$, differential of $y$, propagation of error, error in measurement, propagated error, relative error, percentage error, marginal cost, marginal revenue, demand function |
| Facts/Rules/Theorems | differential rules (linearity rule, product rule, quotient rule, power rule, trigonometric rules, exponential and logarithmic rules, inverse trigonometric rules) |
| Supplementary Problems | **3.8**: every other odd 1-49 |

**28 Feb 4.1**

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|---|---|
| Critical Ideas | extreme value theorem, relative extrema, absolute extrema, optimization |
| Terms/Definitions | optimization problems, absolute maximum, absolute minimum, absolute extrema, extreme values, relative maximum, relative minimum, relative extrema, critical number of $f$, critical point on the graph of $f$ |
| Facts/Rules/Theorems | **theorem**: extreme value of a continuous function on $[a,b]$, **theorem**: critical number theorem |
| Supplementary Problems | **4.1**: every other odd 1-13; every other odd 21-57 |

**05 Mar 4.2**

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|---|---|
| Critical Ideas | Rolle's theorem, statement and proof of the mean value theorem, the zero-derivative theorem |
| Terms/Definitions | |
| Facts/Rules/Theorems | **theorem**: Rolle's theorem, **theorem**: mean value theorem, **theorem**: zero-derivative theorem, **theorem**: constant difference theorem |
| Supplementary Problems | **4.2**: every other odd 5-41 |

**07-09 Mar 4.3**

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|---|---|
| Critical Ideas | increasing and decreasing functions, the first-derivative test, concavity and inflection points, the second derivative test, curve sketching using the first and second derivatives |
| Terms/Definitions | strictly increasing on an interval, strictly decreasing on an interval, montonic, relative maximum, relative minimum, not an extremum, concave up, concave down, inflection point of a graph, second-order critical number, first-order critical number, diminishing returns |
| Facts/Rules/Theorems | **theorem**: monotone function theorem, first derivative test, second derivative test |
| Supplementary Problems | **4.3**: every other odd 5-49 |

**19-21 Mar 4.4**

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|---|---|
| Critical Ideas | limits to infinity, infinite limits, graphs with asymptotes, vertical tangents and cusps, a general graphing strategy |
| Terms/Definitions | limits to infinity, infinite limits, vertical asymptote, horizontal asymptote, vertical tangent, cusp, extent, symmetry |
| Facts/Rules/Theorems | **theorem**: special limits to infinity ($\lim\limits_{x \to \infty} \dfrac{A}{x^r} = 0,\ \text{for } r > 0$) |

| Supplementary Problems | **4.4**: every other odd 5-41 |
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## 23-26 Mar **4.5**

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| Critical Ideas | a rule to evaluate indeterminate forms, indeterminate forms 0/0 and $\infty/\infty$, other indeterminate forms, special limits involving $e^x$ and $\ln x$ |
| Terms/Definitions | indeterminate forms |
| Facts/Rules/Theorems | **theorem**: l'Hopital's rule (0/0, $\infty/\infty$), other indeterminate forms ($1^\infty$, $0 \bullet \infty$, $0^0$, $\infty^0$, $\infty - \infty$), **theorem**: limits involving exponentials and logarithms ( $\lim\limits_{x\to 0^+}\dfrac{\ln x}{x^n} = -\infty$, $\lim\limits_{x\to\infty}\dfrac{\ln x}{x^n} = 0$, $\lim\limits_{x\to\infty}\dfrac{e^{kx}}{x^n} = \infty$ ) |
| Supplementary Problems | **4.5**: every other odd 1-53 |

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## 28 Mar **4.6**

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| Critical Ideas | optimization procedure, Fermat's principle of optics and Snell's law |
| Terms/Definitions | optimization problems, optimization, evt convention, Fermat's principle of optics, Snell's law of refraction, relative index of refraction |
| Facts/Rules/Theorems | |
| Supplementary Problems | **4.6**: 7, 9, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 24, 26 |

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## **4.7**

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| Critical Ideas | economics: maximizing profit and marginal analysis, business management: an inventory model and optimal holding time, physiology: concentration of a drug in the bloodstream and optimal angle for vascular branching |
| Terms/Definitions | discrete functions, marginal analysis, demand function, total revenue, total profit, marginal cost, marginal revenue, average cost, Poiseuille's resistance to flow law |
| Facts/Rules/Theorems | |
| Supplementary Problems | **4.7**: every other odd 1-29 |

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## 02 Apr **5.1**

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| Critical Ideas | reversing differentiation, antiderivative notation, antidifferentiation formulas, applications, area as an antiderivative |
| Terms/Definitions | antiderivative, slope field, direction field, indefinite integral of $f$, indefinite integration, constant of integration, area function |
| Facts/Rules/Theorems | **theorem**: any two antiderivatives of a function differ by a constant, **theorem**: basic integration rules [constant multiple rule, sum rule, difference rule, linearity |

rule, constant rule ( $\int 0 \, du = 0 + c$ ), power rule ( $\int u^n \, du = \dfrac{u^{n+1}}{n+1} + c, \; n \neq -1$ ),

exponential rule ( $\int e^u \, du = e^u + c$ ), logarithm rule, ( $\int \dfrac{1}{u} \, du = \ln |u| + c$ ),

trigonometric rules, inverse trigonometric rules], **theorem**: area as an antiderivative

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| Supplementary Problems | **5.1**: odd 1-29; odd 41-51 |

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06 Apr **5.2**

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|---|---|
| Critical Ideas | area as the limit of a sum, the general approximation scheme, summation notation, area using summation notation |
| Terms/Definitions | summation notation, sigma notation, index of summation, dummy variable |
| Facts/Rules/Theorems | **theorem**: basic rules for summation (constant term rule, sum rule, scalar multiple rule, linearity rule, dominance rule) |
| Supplementary Problems | **5.2**: odd 1-27; odd 39-43 |

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11 Apr **5.3**

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|---|---|
| Critical Ideas | Riemann sums, the definite integral, area as an integral, properties of the definite integral, distance as an integral |
| Terms/Definitions | partition, Riemann sum, norm of partition, regular partition, $f$ is integrable on $[a,b]$, definite integral of $f$ from $a$ to $b$, integrand, interval of integration, lower limit of integration, upper limit of integration, total distance traveled, net distance, net displacement |
| Facts/Rules/Theorems | **theorem**: integrability of a continuous function $f$ on $[a,b]$, **theorem**: properties of definite integrals (linearity rule, dominance rule, subdivision rule) |
| Supplementary Problems | **5.3**: odd 1-29 |

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13 Apr **5.4**

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|---|---|
| Critical Ideas | the first fundamental theorem of calculus, the second fundamental theorem of calculus |
| Terms/Definitions | dummy variable |
| Facts/Rules/Theorems | **theorem**: the first fundamental theorem of calculus [if $F' = f$ on $[a,b]$, then $\int_a^b f(x) \, dx = F(b) - F(a)$ ], **theorem**: the second fundamental theorem of calculus [let $G(x) = \int_a^x f(t) \, dt$ for $x \in [a,b]$, then $G'(x) = f(x)$ ] |
| Supplementary Problems | **5.4**: every other odd 1-57 |

**16-18 Apr 5.5**

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|---|---|
| Critical Ideas | substitution with indefinite integrals, substitution with definite integrals |
| Terms/Definitions | dummy variable |
| Facts/Rules/Theorems | |
| Supplementary Problems | **5.5**: every other odd 1-41 |

**20 Apr 5.6**

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|---|---|
| Critical Ideas | introduction and terminology, direction fields, separable differential equations, modeling exponential growth and decay, orthogonal trajectories, modeling fluid flow through an orifice, modeling the motion of a projectile: escape velocity |
| Terms/Definitions | differential equation, solution, general solution, solved, slope field, direction field, separable, exponential change, growth, decay, carbon dating, orthogonal trajectory, isotherms, velocity potential curves, escape velocity |
| Facts/Rules/Theorems | |
| Supplementary Problems | **5.6**: every other odd 1-33; odd 43-51 |

**23 Apr 5.7**

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|---|---|
| Critical Ideas | mean value theorem for integrals, modeling average value of a function |
| Terms/Definitions | average value, trapezoid rule, Simpson's rule, natural logarithm, inversion formulas |
| Facts/Rules/Theorems | **theorem**: mean value theorem for integrals [ $\int_a^b f(x)\,dx = f(c)\,(b-a)$ for some $c \in (a,b)$ ], average value of $f$ on [$a,b$] is $\dfrac{1}{b-a}\int_a^b f(x)\,dx$ |
| Supplementary Problems | **5.7**: every other odd 1-33 |

**25 Apr 5.8**

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|---|---|
| Critical Ideas | approximation by rectangles, trapezoid rule, Simpson's rule, error estimation |
| Terms/Definitions | |
| Facts/Rules/Theorems | trapezoid rule, Simpson's rule, error estimate in trapezoid rule ( $\mid E \mid \le \dfrac{(b-a)^3}{12n^2} M$ ), error estimate in Simpson's rule ( $\mid E \mid \le \dfrac{(b-a)^5}{180n^4} K$ ) |
| Supplementary Problems | **5.8**: odd 1-25 |

27 Apr **5.9**

Critical Ideas                    natural logarithm as an integral, geometric interpretation, the natural exponential
                                   function

Terms/Definitions

Facts/Rules/Theorems          **theorem**: properties of natural logarithm function defined as $\ln x = \int_1^x \frac{1}{t}\, dt$ ,

                                   properties of exponential function defined as inverse of natural logarithm function

Supplementary Problems        **5.9**: 2, 3