Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Attach this question sheet to the front of your answer sheets.

1. Consider the function \( c(t) = \frac{0.12t}{t^2 + t + 2} \). This function models the concentration of a drug in the bloodstream \( t \) hours after injection of the drug.
   a. (9 pts) Construct a linearization for the function at \( t = 4 \).
   b. (3 pts) Use the linearization constructed in step a. to estimate the change in concentration over the time period from 4 hours after injection to 5 hours after injection.

2. (15 pts) Using calculus, find the absolute maximum and absolute minimum values of \( f(x) = 3x^4 + 4x^3 - 36x^2 + 70 \) on the interval \([-4,1]\).

3. (45 pts) Consider the function \( f \) given below. (Its derivatives \( f' \) and \( f'' \) are denoted by \( f_p \) and \( f_pp \), resp.) Find and identify each of the following (if they exist) – show your work on your answer sheets, but record your solutions to parts a. through m. on the back of this page.
   a. domain of \( f \)
   b. intercepts of \( f \)
   c. vertical asymptotes to the graph of \( f \)
   d. horizontal asymptotes to the graph of \( f \)
   e. critical numbers of \( f \)
   f. intervals on which the graph of \( f \) is increasing
   g. intervals on which the graph of \( f \) is decreasing
   h. local maximum points of the graph of \( f \)
   i. local minimum points of the graph of \( f \)
   j. 2nd order critical numbers of \( f \)
   k. intervals on which the graph of \( f \) is concave up
   l. intervals on which the graph of \( f \) is concave down
   m. inflection points of the graph of \( f \)

   Then, incorporate all of the above information into a sketch the graph of \( f \).

4. (30 pts) Using algebraic/calculus techniques, find the following limits (if they exist):
   a. \( \lim_{x \to \infty} \frac{x^3 + 2\sqrt{x} - 1}{3x^3 - 4x + 1} \)
   b. \( \lim_{x \to 0} \frac{x \sin x}{1 - \cos x} \)
   c. \( \lim_{x \to 0} \frac{2x + \sin x}{2x - \cos x} \)
   d. \( \lim_{x \to \infty} x^2 e^{-x} \)
   e. \( \lim_{x \to \infty} (x + 1)^{\frac{1}{(x+1)}} \)

Extra Credit: Using the rules developed in class for differentiation, correctly derive the formulas given in Problem 3 for \( f' \) and \( f'' \) (2 pts and 3 pts, resp.).
## Solution Space for Problem 3

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