Exam III - A
Key

1a) $f(4) = \sqrt{2(4)+1} = 3$

$$f'(x) = \frac{1}{2\sqrt{2x+1}}\, 2 = \frac{1}{\sqrt{2x+1}}$$

$$f'(4) = \frac{1}{\sqrt{2(4)+1}} = \frac{1}{3}$$

$$L(x) = f(4) + f'(4)(x-4) = 3 + \frac{1}{3}(x-4)$$

1b) $\sqrt{8.8} = \sqrt{2(3.9)+1} = f(3.9) \approx L(3.9) = 3 + \frac{1}{3}(3.9-4) = 2.966\overline{6}$

2. $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1)$

C.N. $x = -1, 0, \cancel{2}$

on $[-2, 1]$

C.P $\begin{cases} f(-1) = -4 \\ f(0) = 1 \end{cases}$

E.P $\begin{cases} f(-2) = 33 & \leftarrow \text{abs. max value} \\ f(1) = -12 & \leftarrow \text{abs. min value} \end{cases}$

3. $f'(x) = -\dfrac{6x-1}{(2x-1)^3}$

C.N. $x = \frac{1}{6}, \frac{1}{2}$

		$\frac{1}{6}$		$\frac{1}{2}$	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

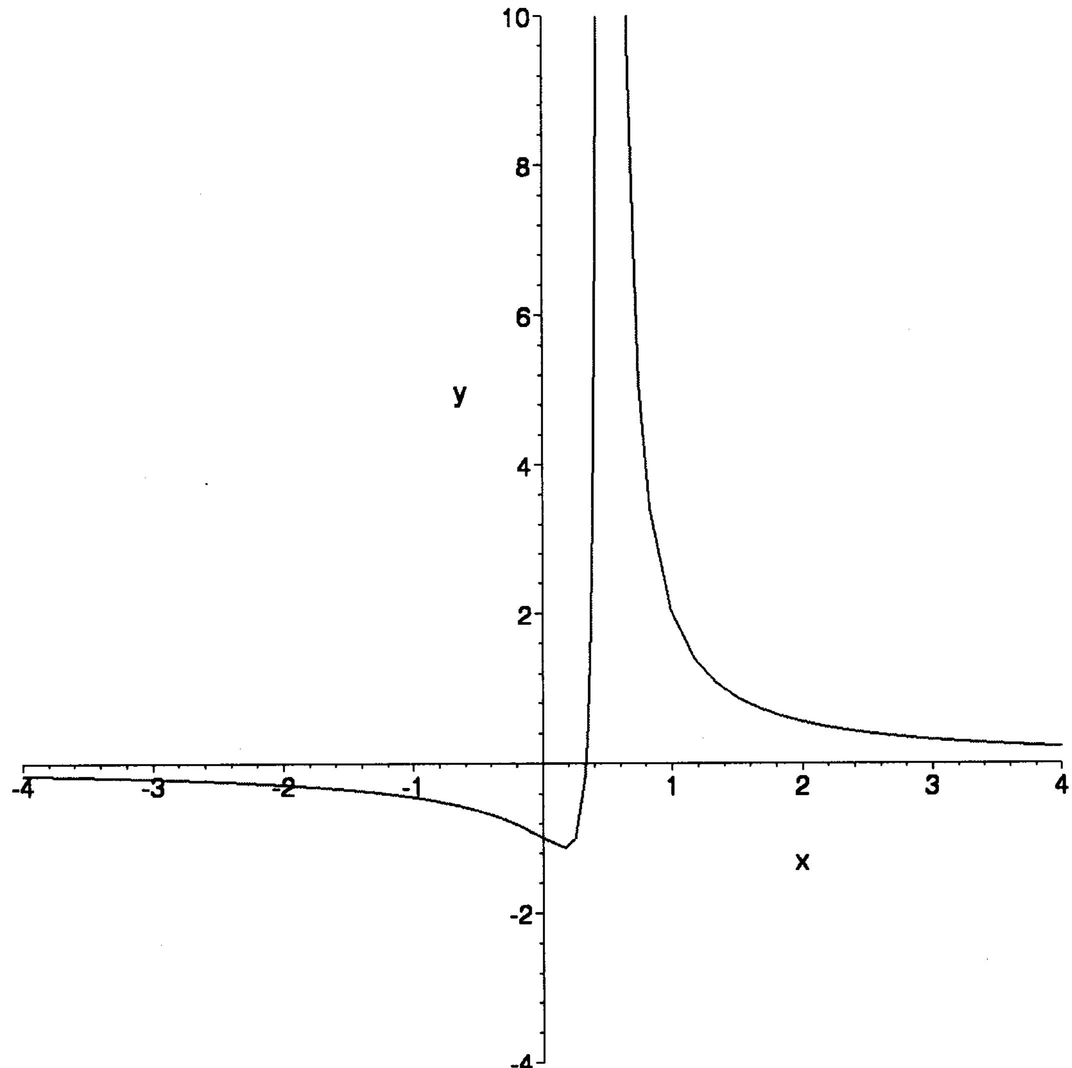
$f''(x) = \dfrac{24x}{(2x-1)^4}$

2nd C.N. $x = 0, \frac{1}{2}$

		0		$\frac{1}{2}$	
f''	$-$		$+$		$+$
f	$\cap$		$\cup$		$\cup$

Solution Space for Problem 3

	Problem Statement	Problem Solution
a	domain of f	all $x \neq \frac{1}{2}$
b	intercepts of f	y $(0,-1)$ x $(\frac{1}{3},0)$
c	vertical asymptotes to the graph of f	$\lim_{x\to\frac{1}{2}^+} f(x) = \infty$, $\lim_{x\to\frac{1}{2}^-} f(x) = \infty$ $\Rightarrow$ $x=\frac{1}{2}$
d	horizontal asymptotes to the graph of f	$\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x)=0$ $\Rightarrow$ $y=0$
e	critical numbers of f	$\frac{1}{6}, \frac{1}{2}$
f	intervals on which the graph of f is increasing	$(\frac{1}{6}, \frac{1}{2})$
g	intervals on which the graph of f is decreasing	$(-\infty, \frac{1}{6}) \cup (\frac{1}{2}, \infty)$
h	local maximum points of the graph of f	NONE
i	local minimum points of the graph of f	$(\frac{1}{6}, -\frac{9}{8})$
j	2nd order critical numbers of f	$0, \frac{1}{2}$
k	intervals on which the graph of f is concave up	$(0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$
l	intervals on which the graph of f is concave down	$(-\infty, 0)$
m	inflection points of the graph of f	$(0,-1)$



4a $$\lim_{x\to\infty} \frac{x^2+2\sqrt{x}-2}{2x^2-4x+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{2}{x^{3/2}}-\frac{2}{x^2}}{2-\frac{4}{x}+\frac{1}{x^2}} = \frac{1}{2}$$

4b $$\lim_{x\to 0} \frac{1-\cos x}{\sin^2 x} \overset{l'H}{=} \lim_{x\to 0} \frac{\cancel{\sin x}}{2\cancel{\sin x}\cos x} = \frac{1}{2}$$

4c $$\lim_{x\to 0} \frac{x+\sin x}{x-\cos x} = \frac{0+0}{0-1} = 0$$

4d $$\lim_{x\to 0^+} x^2 \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} \overset{l'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+} -\frac{x^2}{2} = 0$$

4e $$y = x^{\frac{1}{\sqrt{x}}}$$
$$\ln y = \frac{1}{\sqrt{x}} \ln x = \frac{\ln x}{\sqrt{x}}$$
$$\left.\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \overset{l'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0 \right\} \Rightarrow \lim_{x\to\infty} x^{\frac{1}{\sqrt{x}}} = e^0 = 1$$