Key II-A

1. $f'(x) = \lim_{h\to 0} \dfrac{\left(3(x+h)^2 - 4(x+h)\right) - \left(3x^2 - 4x\right)}{h} = \lim_{h\to 0} \dfrac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 + 4x}{h}$

$= \lim_{h\to 0} \dfrac{h(6x + 3h - 4)}{h} = 6x - 4$

2. $a'(x) = 10x - \frac{8}{3}x^{\frac{1}{3}} + \dfrac{21}{x^4}$

$b'(x) = \dfrac{(x^2+2x)\,4 - (4x-3)(2x+2)}{(x^2+2x)^2} = \dfrac{-4x^2 + 6x + 6}{(x^2+2x)^2}$

$c'(x) = -4\left(x(-4e^{-4x}) + e^{-4x}\right) + 4e^{-4x}$

$= 16x e^{-4x}$

$d'(x) = \dfrac{2x}{1+x^4} - 2\tan x \sec^2 x$

$e'(x) = \frac{1}{2}\dfrac{1}{x^2+2x}(2x+2) = \dfrac{x+1}{x^2+2x}$

3. $f'(x) = x^2(-\sin 2x)2 + 2x\cos 2x$

$f''(x) = x^2(-\cos 2x)2\cdot 2 + 2x(-\sin 2x)2 + 2x(-\sin 2x)2 + 2\cos 2x$

$= -4x^2\cos 2x - 8x\sin 2x + 2\cos 2x$

4. $f'(x) = \dfrac{1}{2\sqrt{x+3}} - 2$

$m = \frac{1}{4} - 2 = -\frac{7}{4}$

$P = (1, 0)$

$y - 0 = -\frac{7}{4}(x-1)$

Key II-A

5. $v(t) = 6 - 4t$

V	+	−
s	advance	retreat

(sign change at $\frac{3}{2}$)

a) $[0, \frac{3}{2})$

b) $(\frac{3}{2}, 4]$

c) $TD = |s(\frac{3}{2}) - s(0)| + |s(4) - s(\frac{3}{2})|$

$= 4\frac{1}{2} + 12\frac{1}{2} = 17$

6. $h(t) = s_0 + v_0 t - 16t^2$

$v(t) = v_0 - 32t$

@ $t = 2$ $v(t) = 0 \Rightarrow v_0 = 64$

@ $t = 9$ $h(t) = 0 \Rightarrow s_0 = 720$

@ $t = 2$ $h(t) = \max$

a) 64

b) 720

c) 784

d) -224

7. $2x - 4(xy' + y) - 3y^2 y' = 0$

$2x - 4y = 4xy' + 3y^2 y' = (4x + 3y^2)y'$

$y' = \frac{2x - 4y}{4x + 3y^2}$

8. $2x + 2(2yy') = 5$

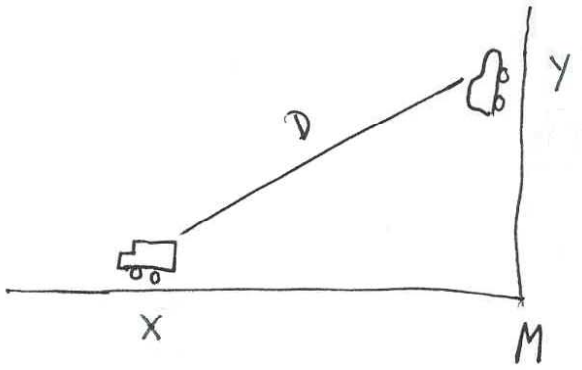
$y' = \frac{5 - 2x}{4y}$

$m = \frac{-1}{4}$

$y - 1 = -\frac{1}{4}(x - 3)$

Key II - A

9.



x = distance truck to M

y = distance car to M

D = distance truck to car

$\frac{dx}{dt} = 50 \qquad \frac{dy}{dt} = 60 \qquad \frac{dD}{dt} = ?$

$D^2 = x^2 + y^2$

$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dD}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{D}$

@ 6^{00} $\quad x = 300$

@ 6^{00} $\quad y = 300$

@ 6^{00} $\quad D = 300\sqrt{2}$

$\frac{dD}{dt} = \frac{300(50) + 300(60)}{300\sqrt{2}} = \frac{110}{\sqrt{2}} \approx 77.78$