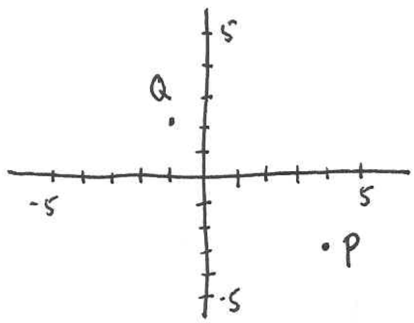
Exam I - A

1.



$$d(P,Q) = \sqrt{(4 - -1)^2 + (-3-2)^2} = \sqrt{50}$$

$$m = \left(\frac{4 + -1}{2}, \frac{-3+2}{2}\right) = \left(\frac{3}{2}, -\frac{1}{2}\right)$$

2. a) $|3-4y| = 5$ $\qquad \left\{-\frac{1}{2}, 2\right\}$

$3 - 4y = 5 \qquad -4y = 2 \qquad y = -\frac{1}{2}$

or

$-(3-4y) = 5 \qquad -3+4y = 5 \qquad 4y = 8 \qquad y = 2$

b) $x^2 + x > 0$ $\qquad \{(-\infty, -1) \cup (0, \infty)\}$

$x(x+1) > 0$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -1 & & 0 & \end{array}$$

3. $x^2 - 4x + \underline{\ 4\ } + y^2 + 6y + \underline{\ 9\ } = 2 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 15 \qquad C = (2, -3), \quad r = \sqrt{15}$

4 $\quad 2y = -3x + 4 \qquad y - 2 = -\frac{3}{2}(x - -3)$

$y = -\frac{3}{2}x + 2$

5. a) $1 + x \geq 0 \qquad x \geq -1 \qquad D = [-1, 0) \cup (0, \infty)$

and

$x \neq 0$

b) $f(-2)$ DNE $\qquad f(-1) = 0 \qquad f(1) = \sqrt{2}$

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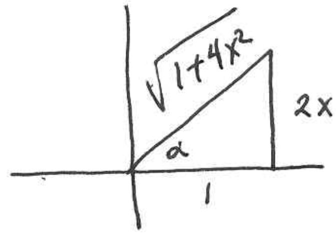
6. $\sin\left(\underset{\alpha}{\tan^{-1} 2x}\right) = \sin\alpha = \dfrac{2x}{\sqrt{1+4x^2}}$

$\tan 2x = \alpha$



7 $\log_2 x(x-15) = 4$

$x(x-15) = 2^4 = 16$

$x^2 - 15x - 16 = 0$

$(x-16)(x+1) = 0$

$x = 16$ or $x = -1$
extraneous

$\{16\}$

9 a) $\displaystyle\lim_{x\to -2} \frac{x^2+3x-4}{x^2-3x+2} = \frac{(-2)^2+3(-2)-4}{(-2)^2-3(-2)+2} = \frac{-6}{12} = -\frac{1}{2}$

b) $\displaystyle\lim_{x\to -2} \frac{4-x^2}{x^2+2x} = \lim_{x\to -2} \frac{\cancel{(2+x)}(2-x)}{x\cancel{(x+2)}} = \frac{4}{-2} = -2$

c) $\displaystyle\lim_{x\to 0} \frac{\sin^2 4x}{x^2 \cos x} = \lim_{x\to 0} \frac{4}{4}\frac{\sin 4x}{x}\,\frac{4}{4}\frac{\sin 4x}{x}\,\frac{1}{\cos x} = 4\cdot 4\cdot 1 = 16$

10. $\displaystyle\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} ax-2 = a-2$

$\displaystyle\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2+2a = 1+2a$

$a-2 = 1+2a \Rightarrow a = -3$

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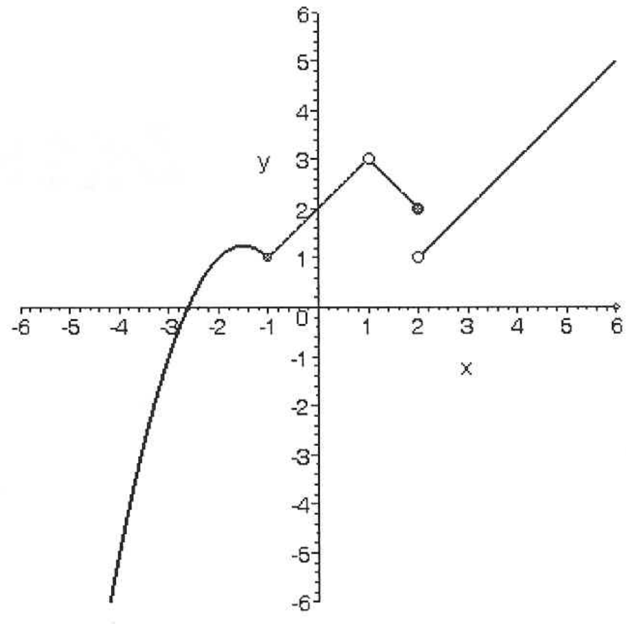
8. (15 pts) Consider the function f defined by the graph to the right. Find each of the following (if they exist). If they do not exist, state so. Also determine if the function is continuous at the given point. If it is not continuous at the given point, state so.



A1. $f(-1)$ = +1

A2. $f(1)$ DNE

A3. $f(2)$ = 2

B1. $\lim_{x\to-1^-} f(x) = +1$

B2. $\lim_{x\to1^-} f(x) = 3$

B3. $\lim_{x\to2^-} f(x) = 2$

C1. $\lim_{x\to-1^+} f(x) = +1$

C2. $\lim_{x\to1^+} f(x) = 3$

C3. $\lim_{x\to2^+} f(x) = 1$

D1. $\lim_{x\to-1} f(x) = +1$

D2. $\lim_{x\to1} f(x) = 3$

D3. $\lim_{x\to2} f(x)$ DNE

E1. Is f continuous at -1? Yes

E2. Is f continuous at 1? No

E3. Is f continuous at 2? No

9. (24 pts) Algebraically evaluate each of the following limits.

A. $\displaystyle\lim_{x\to-2}\frac{x^2+3x-4}{x^2-3x+2}$

B. $\displaystyle\lim_{x\to-2}\frac{4-x^2}{x^2+2x}$

C. $\displaystyle\lim_{x\to0}\frac{\sin^2 4x}{x^2\cos x}$

10. (8 pts) Find the constant a so that $f(x)=\begin{cases} ax-2 & x<1 \\ x^2+2a & x\ge1 \end{cases}$ will be continuous at $x = 1$.