Math 1351-013
Spring 2004

Lecture Summaries

15 Jan **1.1**

Critical Ideas	distance on a number line, absolute value, distance in a plane, trigonometry, solving trigonometric equations
Terms/Definitions	real numbers, real number line, absolute value, distance, interval notation, bounded interval, open interval, half-open interval, closed interval, absolute value equation property, tolerance, absolute error, horizontal change, vertical change, midpoint, analytical geometry, graph of an equation, unit circle, completing the square, degree, radian
Facts/Rules/Theorems	order properties (tricotomy law, transitive law of inequality, additive law of inequality, multiplicative law of inequality), absolute value formula, distance formula on real number line, properties of absolute value, intervals (inequality notation, interval notation, graphical representation), **theorem**: distance formula in the plane, midpoint formula, standard form for the equation of a circle
Supplementary Problems	**1.1**: every other odd 1-45

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20 Jan **1.2/1.3**

Critical Ideas	slope of a line, forms for the equation of a line, parallel and perpendicular lines, definition of a function, functional notation, domain of a function, composition of functions, graph of a function, classification of functions
Terms/Definitions	inclination, slope, angle of inclination, parallel, perpendicular, function, image, domain, range, onto function, one-to-one function, bounded function, variables, dependent variable, independent variable, evaluate, difference quotient, piecewise-defined function, domain convention, undefined, equal functions, hole, composite function, graph, vertical line test, y-intercept, x-intercept, symmetry, symmetric with respect to the y-axis, even function, symmetric with respect to the origin, odd function, polynomial function, degree, leading coefficient, constant term, constant function, linear function, quadratic function, cubic function, quartic function, rational function, power function, algebraic function, transcendental function, trigonometric functions, exponential functions, logarithmic functions
Facts/Rules/Theorems	formula for the slope of a line, formula for the angle of inclination of a line, forms of the equation of a line (standard form, slope-intercept form, point-slope form, two-intercept form, horizontal line, vertical line), slope criteria for parallel and perpendicular lines, rule for equality of two functions, rules for finding the y-intercepts and x-intercept(s) of a function, test for y-axis symmetry of the graph of a function, test for origin symmetry of the graph of a function

Supplementary Problems	**1.2**: every other odd 1-45; **1.3**: every other odd 1-61

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22 Jan **1.4**

Critical Ideas	inverse functions, criteria for existence of an inverse f^{-1}, graph of f^{-1}, inverse trigonometric functions, inverse trigonometric identities
Terms/Definitions	inverse of f, one-to-one function, horizontal line test, strictly increasing, strictly decreasing, strictly monotonic, reference triangle
Facts/Rules/Theorems	**theorem**: a strictly monotonic function has an inverse, procedure for finding the graph of the inverse of a function, graphs of $\sin^{-1} x$, $\tan^{-1} x$, inversion formulas for trigonometric functions
Supplementary Problems	**1.4**: every other odd 5-41

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27 Jan **2.1**

Critical Ideas	informal computation of limits, one-sided limits, limits that do not exist, formal definition of a limit
Terms/Definitions	limit of a function, right-hand limit, left-hand limit, diverge, tend to infinity, divergence by oscillation, epsilon-delta definition
Facts/Rules/Theorems	$\lim_{x\to c} f(x) = L$, $\lim_{x\to c^+} f(x) = L$, $\lim_{x\to c^-} f(x) = L$, **theorem**: one-side limit theorem, $\lim_{x\to c} f(x) = +\infty$, $\lim_{x\to c} f(x) = -\infty$
Supplementary Problems	**2.1**: every other odd 1-41

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29 Jan **2.2**

Critical Ideas	computations with limits, using algebra to find limits, limits of piecewise-defined functions, two special trigonometric limits
Terms/Definitions	squeeze rule
Facts/Rules/Theorems	basic properties and rules for limits (constant rule, limit of x rule, multiple rule, sum rule, difference rule, product rule, quotient rule, power rule), **theorem**: limit of a polynomial function, **theorem**: limit of a rational function (where defined), **theorem**: limits of trigonometric functions (where defined), **theorem**: special limits ($\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$)
Supplementary Problems	**2.2**: every other odd 1-57

03 Feb **2.3**

Critical Ideas	intuitive notion of continuity, definition of continuity, continuity theorems, continuity on an interval, the intermediate value theorem
Terms/Definitions	continuous at a point $x=c$, discontinuity, continuous from the right at a, continuous from the left at a, continuous on the open interval (a,b), continuous on the half-open interval $[a,b)$, continuous on the half-open interval $(a,b]$, continuous on the closed interval $[a,b]$, suspicious point, intermediate value property, root
Facts/Rules/Theorems	**theorem**: continuity theorem (polynomials, rational functions, trigonometric functions, inverse trigonometric functions are continuous (where defined)), **theorem**: properties of continuous functions (scalar multiples, sums and differences, products, quotients (where defined), compositions (where defined) of continuous functions are again continuous functions), **theorem**: intermediate value theorem, **theorem**: root location theorem
Supplementary Problems	**2.3**: every other odd 1-41

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05 Feb **2.4**

Critical Ideas	exponential functions, logarithmic functions, natural exponential and logarithmic functions, continuous compounding of interest
Terms/Definitions	completeness property, exponential function with base b, logarithm of x to the base b, exponent to the base b, natural exponential base, natural exponential function, natural logarithm, common logarithm, continuous compounding of interest, present value, principal, interest rate, future value
Facts/Rules/Theorems	**theorem**: properties of exponential functions (equality rule, inequality rules, product rule, quotient rule, power rules), **theorem**: properties of logarithmic functions (equality rule, inequality rules, product rule, quotient rule, power rule, inversion rules, special values), **theorem**: basic properties of natural logarithm ($\ln 1 = 0$, $\ln e = 1$, $e^{\ln x} = x$, $\ln e^{y} = y$, $b^{x} = e^{x \ln b}$), **theorem**: change of base ($\log_b x = \frac{\ln x}{\ln b}$)
Supplementary Problems	**2.4**: every other odd 1-61

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10 Feb **3.1**

Critical Ideas	tangent lines, the derivative, relationship between the graphs of f and f', existence of derivatives, continuity and differentiability, derivative notation
Terms/Definitions	secant line, slope of tangent line, difference quotient, derivative of f, differentiate f at x, f differentiable at x
Facts/Rules/Theorems	formula for the slope of a tangent line to $y = f(x)$ at $x = x_0$, formula for the derivative

of a function f at x ($\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = f'(x)$), **theorem**: formula for the equation of a tangent line to $y = f(x)$ at $x = x_0$, **theorem**: differentiability implies continuity,

Supplementary Problems — **3.1**: every other odd 5-61

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12 Feb **3.2**

Critical Ideas — derivative of a constant function, derivative of a power function, procedural rules for finding derivatives, higher-order derivatives

Terms/Definitions — first derivative of f, second derivative of f, third derivative of f, nth derivative

Facts/Rules/Theorems — **theorem**: constant rule, **theorem**: power rule, **theorem**: basic procedural rules (constant multiple, sum rule, difference rule, linearity rule, product rule, quotient rule)

Supplementary Problems — **3.2**: every other odd 1-49

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17 Feb **3.3**

Critical Ideas — derivatives of the sine and cosine functions, differentiation of the other trigonometric functions, derivatives of exponential and logarithmic functions

Terms/Definitions

Facts/Rules/Theorems — **theorem**: trigonometric functions ($(\sin x)' = \cos x$, $(\cos x)' = -\sin x$), **theorem**: other trigonometric functions, **theorem**: natural exponential function ($(e^x)' = e^x$), **theorem**: natural logarithm function ($(\ln x)' = \frac{1}{x}$)

Supplementary Problems — **3.3**: every other odd 1-53

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19 Feb **3.4**

Critical Ideas — average and instantaneous rate of change, introduction to mathematical modeling, rectilinear motion (modeling in physics), falling body problem

Terms/Definitions — average rate of change of y with respect to x, instantaneous rate of change, relative rate of change, mathematical modeling, abstraction, velocity, acceleration, speed, advancing, retreating, accelerating, decelerating, position, falling body problem

Facts/Rules/Theorems

Supplementary Problems — **3.4**: every other odd 5-61

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24 Feb **3.5**

Critical Ideas	introduction to the chain rule, extended derivative formulas, justification of the chain rule
Terms/Definitions	horizontal tangent line
Facts/Rules/Theorems	**theorem**: chain rule ($[f(g(x))]' = f'(g(x))g'(x)$), extended power rule ($[u^n]' = nu^{n-1}u'$), extended trigonometric rules, extended exponential and logarithmic rules
Supplementary Problems	**3.5**: every other odd 5-61

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26 Feb **3.6**

Critical Ideas	general procedure for implicit differentiation, derivative formulas for the inverse trigonometric functions, logarithmic differentiation
Terms/Definitions	explicitly defined function, implicitly defined function, implicit differentiation, logarithmic differentiation
Facts/Rules/Theorems	**theorem**: differentiation rules for inverse trigonometric functions, **theorem**: differentiation of exponential and logarithmic functions with base b
Supplementary Problems	**3.6**: every other odd 1-57

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02 Mar **3.7**

Critical Ideas	
Terms/Definitions	related rate problems, general situation, specific situation
Facts/Rules/Theorems	
Supplementary Problems	**3.7**: every other odd 1-45

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04 Mar **3.8**

Critical Ideas	tangent line approximation, the differential, error propagation, marginal analysis in economics, the Newton-Raphson method for approximating roots
Terms/Definitions	linear approximation, linearization, incremental approximation formula, differential of x, differential of y, propagation of error, error in measurement, propagated error, relative error, percentage error, marginal cost, marginal revenue, demand function
Facts/Rules/Theorems	differential rules (linearity rule, product rule, quotient rule, power rule, trigonometric rules, exponential and logarithmic rules, inverse trigonometric rules)
Supplementary Problems	**3.8**: every other odd 1-49

09 Mar **4.1**

Critical Ideas	extreme value theorem, relative extrema, absolute extrema, optimization
Terms/Definitions	optimization problems, absolute maximum, absolute minimum, absolute extrema, extreme values, relative maximum, relative minimum, relative extrema, critical number of f, critical point on the graph of f
Facts/Rules/Theorems	**theorem**: extreme value of a continuous function on $[a,b]$, **theorem**: critical number theorem
Supplementary Problems	**4.1**: every other odd 1-13; every other odd 21-57

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11 Mar **4.2**

Critical Ideas	Rolle's theorem, statement and proof of the mean value theorem, the zero-derivative theorem
Terms/Definitions	
Facts/Rules/Theorems	**theorem**: Rolle's theorem, **theorem**: mean value theorem, **theorem**: zero-derivative theorem, **theorem**: constant difference theorem
Supplementary Problems	**4.2**: every other odd 5-41

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23 Mar **4.3**

Critical Ideas	increasing and decreasing functions, the first-derivative test, concavity and inflection points, the second derivative test, curve sketching using the first and second derivatives
Terms/Definitions	strictly increasing on an interval, strictly decreasing on an interval, montonic, relative maximum, relative minimum, not an extremum, concave up, concave down, inflection point of a graph, second-order critical number, first-order critical number, diminishing returns
Facts/Rules/Theorems	**theorem**: monotone function theorem, first derivative test, second derivative test
Supplementary Problems	**4.3**: every other odd 5-49

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25 Mar **4.4**

Critical Ideas	limits to infinity, infinite limits, graphs with asymptotes, vertical tangents and cusps, a general graphing strategy
Terms/Definitions	limits to infinity, infinite limits, vertical asymptote, horizontal asymptote, vertical tangent, cusp, extent, symmetry

Facts/Rules/Theorems **theorem**: special limits to infinity ($\lim_{x\to\infty}\frac{A}{x^r}=0,\ \text{for } r>0$)

Supplementary Problems **4.4**: every other odd 5-41

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30 Mar **4.5**

Critical Ideas a rule to evaluate indeterminate forms, indeterminate forms 0/0 and ∞/∞, other indeterminate forms, special limits involving e^x and $\ln x$

Terms/Definitions indeterminate forms

Facts/Rules/Theorems **theorem**: l'Hôpital's rule (0/0, ∞/∞), other indeterminate forms (1^∞, $0\cdot\infty$, 0^0, ∞^0, $\infty-\infty$), **theorem**: limits involving exponentials and logarithms ($\lim_{x\to 0^+}\frac{\ln x}{x^n}=-\infty$, $\lim_{x\to\infty}\frac{\ln x}{x^n}=0$, $\lim_{x\to\infty}\frac{e^{kx}}{x^n}=\infty$)

Supplementary Problems **4.5**: every other odd 1-53

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01 Apr **4.6**

Critical Ideas optimization procedure, Fermat's principle of optics and Snell's law

Terms/Definitions optimization problems, optimization, evt convention, Fermat's principle of optics, Snell's law of refraction, relative index of refraction

Facts/Rules/Theorems

Supplementary Problems **4.6**: 7, 9, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 24, 26

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06 Apr **4.7**

Critical Ideas economics: maximizing profit and marginal analysis, business management: an inventory model and optimal holding time, physiology: concentration of a drug in the bloodstream and optimal angle for vascular branching

Terms/Definitions discrete functions, marginal analysis, demand function, total revenue, total profit, marginal cost, marginal revenue, average cost, Poiseuille's resistance to flow law

Facts/Rules/Theorems

Supplementary Problems **4.7**: every other odd 1-29

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08 Apr **5.1**

Critical Ideas reversing differentiation, antiderivative notation, antidifferentiation formulas,

applications, area as an antiderivative

Terms/Definitions — antiderivative, slope field, direction field, indefinite integral of f, indefinite integration, constant of integration, area function

Facts/Rules/Theorems — **theorem**: any two antiderivatives of a function differ by a constant, **theorem**: basic integration rules [constant multiple rule, sum rule, difference rule, linearity rule, constant rule ($\int 0\ du = 0 + c$), power rule ($\int u^n\ du = \frac{u^{n+1}}{n+1} + c,\ n \neq -1$), exponential rule ($\int e^u\ du = e^u + c$), logarithm rule, ($\int \frac{1}{u}\ du = \ln|u| + c$), trigonometric rules, inverse trigonometric rules], **theorem**: area as an antiderivative

Supplementary Problems — **5.1**: odd 1-29; odd 41-51

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13 Apr **5.2**

Critical Ideas — area as the limit of a sum, the general approximation scheme, summation notation, area using summation notation

Terms/Definitions — summation notation, sigma notation, index of summation, dummy variable

Facts/Rules/Theorems — **theorem**: basic rules for summation (constant term rule, sum rule, scalar multiple rule, linearity rule, dominance rule)

Supplementary Problems — **5.2**: odd 1-27; odd 39-43

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15 Apr **5.3**

Critical Ideas — Riemann sums, the definite integral, area as an integral, properties of the definite integral, distance as an integral

Terms/Definitions — partition, Riemann sum, norm of partition, regular partition, f is integrable on $[a,b]$, definite integral of f from a to b, integrand, interval of integration, lower limit of integration, upper limit of integration, total distance traveled, net distance, net displacement

Facts/Rules/Theorems — **theorem**: integrability of a continuous function f on $[a,b]$, **theorem**: properties of definite integrals (linearity rule, dominance rule, subdivision rule)

Supplementary Problems — **5.3**: odd 1-29

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20 Apr **5.4**/**5.5**

Critical Ideas — the first fundamental theorem of calculus, the second fundamental theorem of calculus, substitution with indefinite integrals, substitution with definite integrals

Terms/Definitions — dummy variable

Facts/Rules/Theorems — **theorem**: the first fundamental theorem of calculus [if $F' = f$ on $[a,b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$], **theorem**: the second fundamental theorem of calculus [let $G(x) = \int_a^x f(t)\,dt$ for $x \in [a,b]$, then $G'(x) = f(x)$]

Supplementary Problems — **5.4**: every other odd 1-57; **5.5**: every other odd 1-41

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22 Apr **5.6**

Critical Ideas — introduction and terminology, direction fields, separable differential equations, modeling exponential growth and decay, orthogonal trajectories, modeling fluid flow through an orifice, modeling the motion of a projectile: escape velocity

Terms/Definitions — differential equation, solution, general solution, solved, slope field, direction field, separable, exponential change, growth, decay, carbon dating, orthogonal trajectory, isotherms, velocity potential curves, escape velocity

Facts/Rules/Theorems

Supplementary Problems — **5.6**: every other odd 1-33; odd 43-51

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27 Apr **5.7**

Critical Ideas — mean value theorem for integrals, modeling average value of a function

Terms/Definitions — average value, trapezoid rule, Simpson's rule, natural logarithm, inversion formulas

Facts/Rules/Theorems — **theorem**: mean value theorem for integrals [$\int_a^b f(x)\,dx = f(c)\,(b-a)$ for some $c \in (a,b)$], average value of f on $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$

Supplementary Problems — **5.7**: every other odd 1-33

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29 Apr **5.8/5.9**

Critical Ideas — approximation by rectangles, trapezoid rule, Simpson's rule, error estimation, natural logarithm as an integral, geometric interpretation, the natural exponential function

Terms/Definitions

Facts/Rules/Theorems — trapezoid rule, Simpson's rule, error estimate in trapezoid rule ($|E| \leq \frac{(b-a)^3}{12n^2} M$),

error estimate in Simpson's rule ($|E| \leq \frac{(b-a)^5}{180n^4}K$), **theorem**: properties of natural logarithm function defined as $\ln x = \int_1^x \frac{1}{t}\,dt$, properties of exponential function defined as inverse of natural logarithm function

Supplementary Problems **5.8**: odd 1-25; **5.9**: 2, 3